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Split Domination In Interval-valued Fuzzy Graphs

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

Aims / Objectives: In this paper, we introduced and investigated the concept of split domination in interval-valued fuzzy graph and denoted by γ_s. We obtained many results related to γ_s. We investigated and study the relationship of γ_s with other known parameters in interval-valued fuzzy graph. Finally we calculated $\gamma_s(G)$ for some standard interval valued fuzzy graphs.

Keywords: Fuzzy graph; interval-valued fuzzy graph; split domination number.

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1 Introduction

Fuzzy graph is one of the application tool in the field of mathematics, which allow the users to describe the relationship between any notions easily.

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Zadeh [1] was introduced the notion of interval-valued fuzzy sets as an extension of fuzzy sets [2] in which the values of the membership degrees are intervals of numbers instead of the numbers. Interval-valued fuzzy sets provide a more adequate description of uncertainty than traditional fuzzy sets. The fuzzy relations between fuzzy sets were also considered by Rosenfeld [3] and he developed the structure of fuzzy graphs. Recently, Akram and Dudek [4] have studied several properties and operations on interval-valued fuzzy graphs [5]. Some important works in fuzzy graph theory can be found in [6, 7, 8, 9, 10, 11, 12]. Somasundaram and Somasundaran [13, 14] introduced and investigated the concept of domination in fuzzy graphs, while P. Debnath [15] introduced and investigated the concept of domination in interval-valued fuzzy graphs. In (1997) Kulli [16] introduced and investigated the concept of split domination number of graph then in (2008) Mahyoub and Sonar discussed the split domination number of fuzzy graphs [17]. In (2017) A. Prasanna, C. Gurubaran and S. Ismail Mohideen [18] are introduced and investigated the concept split and nonsplit domination number in bipolar fuzzy graphs. In (2019) R. Muthuraj and A. Kanimozhi [19] are define Split Total strong (weak) domination in bipolar fuzzy graph. Since the theory of fuzzy sets has become a vigorous area of research in different disciplines including medical and life sciences, management sciences, social sciences engineering, statistic, graph theory, artificial intelligence, signal processing, multi-agent systems, decision making and automate theory and also the domination in fuzzy graphs, this is what motivated us to submit this work. In this paper, we will introduce and investigate the concept of split domination in interval-valued fuzzy graphs and we will obtain many result related to this concept. The relationship between this concept and the others in interval-valued fuzzy graph will be given. Finally, we calculated $\gamma_s(G)$ for some standard interval valued fuzzy graphs.

2 Preliminaries

In this section, we review some basic definitions related to interval-valued fuzzy graphs and split domination in fuzzy graph.

The interval-valued fuzzy graph (in short,$IVFG$) of a graph $G^* = (V, E)$ is a pair $G = (A, B)$, where $A = [\mu_A^-, \mu_A^+]$ is an interval-valued fuzzy set on V and $B = [\rho_B^-, \rho_B^+]$ is an interval-valued fuzzy relation on V. In an interval-valued fuzzy graph G, when $\rho^-(u, v) = \rho^+(u, v) = 0$ for some u and v, then there is no edge between u and v. Otherwise, there exists an edge between u and v. Let $G = (A, B)$ be an interval-valued fuzzy graph. Then the cardinality of interval-valued fuzzy graph G is defined as

$$|G| = \sum_{u\in V} \frac{1 + \mu^+(u) - \mu^-(v)}{2} + \sum_{(u,v)\in E} \frac{1 + \rho^+(u, v) - \rho^-(u, v)}{2}.$$

The vertex cardinality of an interval-valued fuzzy graph G is defined by

$$p = \sum_{u\in V} \frac{1 + \mu^+(u) - \mu^-(u)}{2}.$$

For all $u \in V$ is called the order of an interval-valued fuzzy graph and is denoted by $p(G)$. The edge cardinality of an interval-valued fuzzy graph G is defined by

$$q = \sum_{(u,v)\in E} \frac{1 + \rho^+(u, v) - \rho^-(u, v)}{2}.$$

For all $(u, v) \in E$ is called the size of an interval-valued fuzzy graph and is denoted by $q(G)$.

An edge $e = xy$ of an interval-valued fuzzy graph G is called effective edge if $\rho^+(x,y) = min\{\mu^+(x), \mu^+(y)\}$ and $\rho^-(x,y) = min\{\mu^-(x), \mu^-(y)\}$, in this case, the vertex x is called a neighbuor of y and conversely. $N(x) = \{y \in V : y$ is a neighbuor of x$\}$ is called the neighbourhood of x and the degree of a vertex can be generalized in different ways for interval-valued fuzzy graph G. The effective degree of a vertex v in an interval-valued fuzzy graph G is defined to be summation of the weights of the effective edges incident at v and it is denoted by $d_E(v)$. The minimum effective edges degree of G is $\delta_E(G) = min\{d_E(v)|v \in V\}$. The maximum effective degree of G is $\Delta_E(G) = max\{d_E(v)|v \in V\}$.

A vertex u of an interval-valued fuzzy graph G is said to be an isolated vertex if $\rho^-(uv) < min\{\mu^-(u), \mu^-(v)\}$ and $\rho^+(uv) < min\{\mu^+(u), \mu^+(v)\}$ for all $v \in V - \{u\}$ such that there is an edge between u and v, i.e., $N(u) = \phi$.

A set S of vertices of an interval-valued fuzzy graph G is said to be independent if $\rho^-(uv) < min\{\mu^-(u), \mu^-(v)\}$ and $\rho^+(uv) < min\{\mu^+(u), \mu^+(v)\}$ for all $u, v \in S$. The interval-valued fuzzy graph$G$ is said to be complete interval-valued fuzzy graph if $\rho^-(v_i, v_j) = min\{\mu^-(v_i), \mu^-(v_j)\}$, $\rho^+(u,v) = min\{\mu^+(u), \mu^+(v)\}$, for all $u, v \in V$ and denoted by K_p. The complement of an interval-valued fuzzy graph $G = (A, B)$ is an interval-valued fuzzy graph, $\overline{G} = (\overline{A}, \overline{B})$ where
$\overline{\rho^-}(u,v) = min\{\mu^-(u), \mu^-(v)\} - \rho^-(u,v)$ and $\overline{\rho^+}(u,v) = min\{\mu^+(u), \mu^+(v)\} - \rho^+(u,v)$ for all $u, v \in G$. The interval-valued fuzzy graph G of a graph $G^* = (V, E)$ is said to be bipartite if the vertex set V can be partitioned into two non-empty sets V_1 and V_2 such that $\rho^-(xy) = 0$ and $\rho^+(xy) = 0$ if $x, y \in V_1$ or $x, y \in V_2$. Further, if $\rho^-(xy) = min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\rho^+(xy) = min\{\mu_A^+(x), \mu_A^+(y)\}$ for all $x \in V_1$ and $y \in V_2$. Then G is called a complete bipartite interval-valued fuzzy graph is denoted by $K_{\mu_A^-, \mu_A^+}$, where μ_A^- and μ_A^+ are restrictions of μ_A^- and μ_A^+ on V_1 and V_2 respectively. A set of fuzzy vertex which covers all the fuzzy edges is called a fuzzy vertex cover of (G). The minimum fuzzy cardinality of vertex cover sets of an interval-valued fuzzy graph G is called a vertex covering number of G and is denoted by $\alpha(G)$.

Let $G = (A, B)$ be an interval-valued fuzzy graph and let $D \subseteq V$ be a subset of V, then D is called independent set in G if, $\forall u, v \in D$, $\rho_{B^+}(u,v) < \mu_{A^+}(u) \wedge \mu_{A^+}(v)$ and $\rho_{B^-}(u,v) < \mu_{A^-}(u) \wedge \mu_{A^-}(v)$ or $\rho_{B^+}(u,v) = 0$ and $\rho_{B^-}(u,v) = 0$ The maximum fuzzy cardinality of the independent sets in an interval-valued fuzzy graph G is called the independence number of G and is denoted by $\beta(G)$.

Let $G = (A, B)$ be an interval-valued fuzzy graph on V and $x, y \in V$. We say x dominates y if $\rho_B^-(xy) = min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\rho_B^+(xy) = min\{mu_A^+(x), \mu_A^+(y)\}$. A subset D of V is called a dominating set in G if for every $v \in D$, there exists $u \in D$ such that u dominates v. The minimum cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma(G)$. A fuzzy graph $G = (A, B)$ is said to be connected if any two vertices in G are connected.

A dominating set D of a fuzzy graph $G = (\mu, \rho)$ is called split dominating set, if the induced fuzzy subgraph $H = (< V - D >, \mu, \rho)$ is a disconnected.

The minimum fuzzy cardinality of the split dominating set is called the split domination number and is denoted by $\gamma_s(G)$.

The maximum fuzzy cardinality taken over all minimal split dominating set of a fuzzy graph is called the upper split domination of G and is denoted by $\Gamma_s(G)$.

3 Mine Results

Definition 3.1. A split dominating set D of an interval-valued fuzzy graph G is said to be minimal split dominating set if $D - \{v\}$ is not split dominating set of G, $\forall v \in D$.

Definition 3.2. The minimum fuzzy cardinality taken over all minimal split dominating set of an interval-valued fuzzy graph is called the split domination number of G and is denoted by $\gamma_s(G)$. or γ_s.

Definition 3.3. The maximum fuzzy cardinality taken over all minimal split dominating set of an interval-valued fuzzy graph is called the upper split domination number of G and is denoted by $\Gamma_s(G)$.

Definition 3.4. The degree of a vertex v in an interval-valued fuzzy graph G is defined to be sum of the weights of the strong edge incident at v. It is denoted by $d_G(v)$.

The minimum degree of an interval-valued fuzzy graph G is $\delta(G) = min\{d_G(v)|v \in V.\}$

The maximum degree of an interval-valued fuzzy graph G is $\Delta(G) = max\{d_G(v)|v \in V\}$.

Example 3.1. Consider the interval-valued fuzzy graph G given in the Figure 3.1, such that all edges in G are effective.

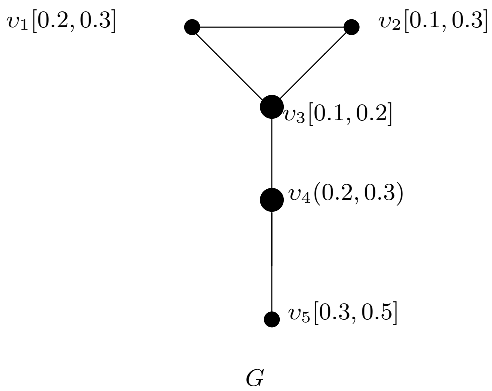


Fig 3.1

From above Figure, we have $D = \{v_3, v_4\}$ is minimal split dominating set. Then $\gamma_s(G) = 0.55 + 0.55 = 1.1$

Remark 3.1. For any interval-valued fuzzy graph G. Then

$$\gamma_s(G) \leq p.$$

Theorem 3.2. *For any interval-valued fuzzy graph G. Then*

$$\gamma_s(G) + \gamma(G) \leq p.$$

Proof. Its obviously true by the definitions ☐

Theorem 3.3. *A split dominating set D of an interval-valued fuzzy graph G is minimal if and only if for each vertex $v \in D$ one of the following conditions holds:*

(i) *There exists a vertex $u \in V - D$ such that $N(u) \cap D = \{v\}$;*
(ii) *v is an isolated in $< D >$;*
(iii) $< V - D^{'} >$ *is connected.*

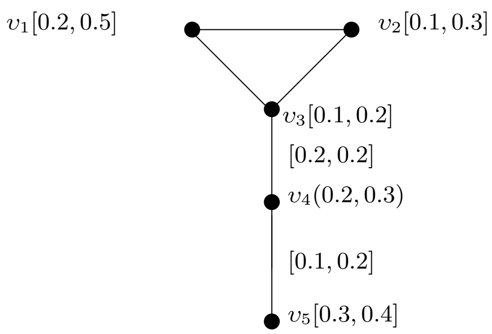
Proof. Let G be an interval-valued fuzzy graph Suppose that D is minimal and there exists a vertex $v \in D$ such that v does not satisfy any of the above conditions. Then by condition (i) and (ii) $D^{'} = D - \{v\}$ is a dominating set of G and also by (iii) $< V - D^{'} >$ is disconnected, this implies $D^{'}$ is split dominating set of G, which is contradiction. □

Theorem 3.4. *Every split dominating set of interval-valued fuzzy graph G is dominating set, but the converse is not true.*

Proof. By Definition (3.1), the theorem follows □

The following example discusses the converse of the above Theorem.

Example 3.5. Consider the interval-valued fuzzy graph G given in the Figure 3.2, such that all edges in G are effective except $v_4v_5 = [0.1, 0.2], v_3v_4 = [0.2, 0.2]$.



G

Fig 3.2

From above Example, we have $\gamma(G) = |v_3, v_4, v_5| = 1.65$, *it's minimal dominating set but it's not split dominating set.*

Theorem 3.6. *If $G = (A, B)$ be an independent interval-valued fuzzy graph. Then*

$$\gamma_s(G) = 0.$$

Proof. Let G be an independent interval-valued fuzzy graph G has no split dominating set. Then $D = \phi$. Hence

$$\gamma_s(G) = |\phi| = 0.$$

□

Theorem 3.7. *For any interval-valued fuzzy graph G, without isolated vertex,*

$$\gamma(G) \leq \gamma_s(G).$$

Proof. Let D is a split dominating set of an interval-valued fuzzy graph G.

Then D is a dominating set of G.
Hence

$$\gamma(G) \leq |D| \leq \gamma_s(G).$$

□

Theorem 3.8. *A dominating set D of interval-valued fuzzy graph G is a split dominating set if and only if there exists two interval-valued fuzzy vertices $u, v \in V - D$ such that every $u - v$ path contains an interval-valued fuzzy vertex of D.*

Proof. Suppose that D is a split dominating set of interval-valued fuzzy graph G. Then by Definition the interval-valued fuzzy subgraph $(< V - D >, \mu^{'}, \rho^{'})$ induced by $V - D$ is disconnected. Hence there exists $u, v \in V - D$ such that every $u - v$ path contains a vertex of D. Let $u, v \in V - D$ such that every $u - v$ path contains a vertex of D. Then $V - D$ is disconnected and a fuzzy subgraph induced by $V - D$ is disconnected. Hence D is a split dominating set of G. □

Theorem 3.9. *If $G = P_n$ is a interval-valued fuzzy path, with $n \geq 3$ vertices, such that every $(u_i, u_{i+1}) \in E; \forall i = 1, 2, ..., n - 1$. is effective edge. Then*

$$\gamma_s(G) = min\{p_1, p_2\}.$$

such that $p_1 = |V_1|, p_2 = |V_2|$, V_1 is the set odd-vertices and V_2 is the set even-vertices.

Proof. Let $G = P_n$ is a interval-valued fuzzy path, with $n \geq 3$ vertices. and Since any interval-valued fuzzy path is a bipartite interval-valued fuzzy graph. Then G have not an isolated vertices, $V = V_1 \cup V_2$ and $\rho_B(u, v) = 0; \forall u, v \in V_1 or u, v \in V_2$. Now, since $V_2 = V - V_1$ and $V_1 = V - V_2$ and by Theorem (3.6), there exists two interval-valued fuzzy vertices $u, v \in V - V_1$ such that every $u - v$ path contains an interval-valued fuzzy vertex of V_1. Then $V_2 = V - V_1$ is disconnected. Therefore, V_1 is split dominating set, so $\gamma_s = |V_1| = p_1$. Similarly, $u, v \in V - V_2$. Therefore V_1 is split dominating set. Then $\gamma_s = |V_2| = p_2$. Hence

$$\gamma_s(G) = min\{p_1, p_2\}.$$

□

In the following results, we give γ_s for some standard Interval-valued fuzzy graphs.

Theorem 3.10. *For any complete bipartite interval-valued fuzzy graph $G = K_{n,m}$ such that $m = |v_2|, n = |v_1|$. Then*

$$\gamma_s(G) = min(n, m).$$

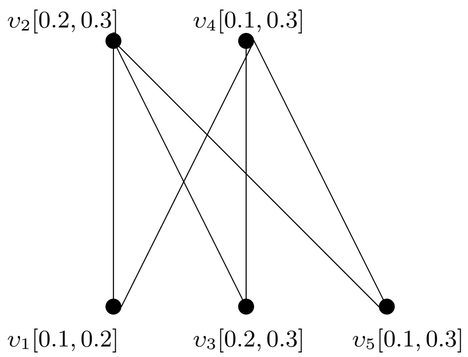
Proof. Let $G = K_{n.m}$ be any complete bipartite interval-valued fuzzy graph. Then $V(G) = V_1 \cup V_2$ such that V_1 and V_2 are independent and $< V - V_1 >$ and $< V - V_2 >$ are disconnected and $\forall v \in V_2, u \in V_1$. Then uv is defective edge. Therefore, G has only two split dominating set, $D_1 = V_1$ and $D_2 = V_2$. Hence

$$\gamma_s(G) = min\{|V_1|, |V_2|\}.$$

□

The following example discusses in detail the result in the above Theorem.

Example 3.11. Consider the interval-valued fuzzy graphs G given in the Figure 3.3 such that all edges in G are effective.



$Fig.3.3$

From Figure (3.3), we have $D_1 = \{v_2, v_4\}$ *and* $D_2 = \{v_1, v_3, v_5\}$. *Hence* $\gamma_s = min\{|D_1|, |D_2|\} = |D_1| = 1.15$

Theorem 3.12. *If* $G = K_{1,m}$ *is a star interval-valued fuzzy graph. Then* $\gamma_s(G) = |u|$ *such that* u *is a root vertex.*

Proof. Let $G = K_{1,m}$ be a star interval-valued fuzzy graph then $V(G) = \{u\} \cup V_2$, such that u is a root vertex, $V_2 = \{V_1, ...V_(n-1)\}$ is an independent vertex set.
Since u is a strong neighborhood to all the others vertices.
Then u dominates $V_i, i = 1, ..., (n-1)$ and since V_2 is an independent.
Then $< V_2 >$ is a disconnected interval-valued fuzzy subgraph of G.
Hence $D = \{u\}$ is a split dominating set.

$$\gamma_s(K_{1,m}) = |u|$$

□

Theorem 3.13. *For any interval-valued fuzzy graph* G *without isolated vertices if* D *is an independent and* G *has no a root vertex if* D *is split dominating set of an interval-valued fuzzy graph* G. *Then* $V - D$ *is split dominating set.*

Proof. Let G be an interval-valued fuzzy graph without isolated and D is an independent split dominating set. Then $< V - D >$ is disconnected and $V - D$ is dominating set of G. Since D is an independent set of (G). Then $< D >$ is disconnected and $V - D$ is a split dominating set. □

Theorem 3.14. *For any interval-valued fuzzy graph* G. *Then*

$$\gamma_s \leq p - \Delta_N. \quad and \quad \gamma_s \leq p - \delta_E.$$

Proof. Let G be an interval-valued fuzzy graph and let v be a vertex in G with $d_N(v) = |N(v)| = \Delta_N$. Then $V - N(v)$ is split dominating set and $\bar{D} \subseteq V - N(v)$, then $|\bar{D}| \leq |V - N(v)| = p - |N(v)|$.

Hence

$$\gamma_s \leq p - \Delta_N \leq p - \delta_E.$$

□

Corollary 3.15. *For any interval-valued fuzzy graph G. Then*

$$\gamma_s \leq p - \Delta_E \quad \textit{and} \quad \gamma_s \leq p - \delta_N.$$

Proof. Since $\Delta_E \leq \Delta_N$ for any interval-valued fuzzy graph G. Therefore, by above Theorem, $\gamma_s \leq p - \Delta_N$. Hence

$$\gamma_s \leq p - \Delta_E.$$

□

Remark 3.2. For any interval-valued fuzzy graph G. Then

$$1 - \Delta_N + \Delta_N^{-} \leq p - 1.$$

$$2 - \Delta_N = \Delta_N^{+} + \Delta_N^{-}.$$

Theorem 3.16. *For any interval-valued fuzzy graph G. Then*

$$\gamma_s(G) \leq \alpha(G)$$

such that α is the vertex covering number.

Proof. Let S be a vertex covering set. Then $V - S$ is an independent set and $< V - S >$ is disconnected. S is split dominating set. Then $\gamma_s \leq |S| = \alpha$. Hence

$$\gamma_s \leq \alpha.$$

□

Corollary 3.17. *For any interval-valued fuzzy graph G. Then*

$$\gamma_s \leq p - \beta(G).$$

Remark 3.3. For any interval-valued fuzzy graph G. Then

$$\Delta_N^{+} + \Delta_N^{-} \leq p - 1.$$

Theorem 3.18. *For any interval-valued fuzzy graph G. Then*

$$\gamma_s(G) \leq \frac{p+1}{2}.$$

Proof. since $\gamma_s(G) \leq p - \Delta_N^{+}$ and $\gamma_s(G) \leq p - \Delta_N^{-}$. Then

$$2\gamma_s(G) \leq 2p - (\Delta_N^{+} + \Delta_N^{-}).$$

$$2\gamma_s(G) \leq 2p - (p-1) = p + 1.$$

Hence,

$$\gamma_s(G) \leq \frac{p+1}{2}.$$

□

Definition 3.5. $< G - v >$ is a subgraph of G obtained be deleting the vertex v and $< G - e >$ is a subgraph of G obtained be deleting the edge e.

Definition 3.6. Let G be any interval-valued fuzzy graph, we called $H = (\overline{A}, \overline{B})$ be a subgraph of G if $\overline{V} \subseteq V$ and $\overline{E} \subseteq E$.
1/ If $\overline{V} = V$, then H is called spanning subgraph of G.
2/ If $\overline{V} \subset V$, then H is called partial subgraph of G.

Remark 3.4. For any interval-valued fuzzy graph G. Then

$$1 - \gamma_s(G) \leq \gamma_s(G - v) \qquad \forall v \in G.$$

$$2 - \gamma_s(G) \leq \gamma_s(G - e); \qquad \text{such that} \quad e \text{ is strong edge.}$$

Theorem 3.19. *Let $G = (A, B)$ be any interval-valued fuzzy graph and $H = (a, b)$ is a partial interval-valued fuzzy subgragh of G. Then*

$$\gamma_s(H) \leq \gamma_s(G).$$

Proof. Let G be an interval-valued fuzzy graph with $\gamma(G)$ and H be partial interval-valued fuzzy subgragh of G. with $\gamma(H)$. Then

$$\gamma(G) \leq \gamma(H).$$

Therefore,

$$p - \gamma(G) \geq p - \gamma(H)$$

Hence

$$\gamma_s(H) \leq \gamma_s(G)$$

□

Theorem 3.20. *Let $G = (A, B)$ be any interval-valued fuzzy graph and $H = (a, b)$ is a spanning interval-valued fuzzy subgragh of G. Then*

$$\gamma_s(H) \geq \gamma_s(G).$$

Proof. By Definition (3.6) and Remark(3.14) we get Theorem. □

Theorem 3.21. *For any interval-valued fuzzy graph G. Then*

1/ $\gamma_s(G) \geq \frac{p}{\Delta_N+1}$.

2/$\gamma_s(G) \geq \frac{\Delta_N - p}{\Delta_N+1}$.

3/$\gamma_s(G) \leq \frac{p\Delta_N}{\Delta_N+1}$.

Proof. 1- Since $\gamma \geq \frac{P}{\Delta_N+1}$ and $\gamma \leq \gamma_s$. Hence we get $\gamma_s(G) \geq \frac{P}{\Delta_N+1}$.
2- From 1, we get 2
3- Let D be a split dominating set. Since D is minimal, by Theorem (3.1), it follows that each $v \in D$, there exists $u \in V - D$ such that $0 \leq \rho(u, v) = \mu(u) \wedge \mu(v)$. This implies that $V - D$ is a dominating set of G. Thus $\gamma(G) \leq |V - D| \leq p - \gamma_s(G)$ and by (1)$\gamma_s(G) \geq \frac{p}{\Delta_N+1}$. Hence

$$\gamma_s(G) \leq \frac{p\Delta_N}{\Delta_N + 1}.$$

□

4 Conclusions

In this paper, the split domination number is defined on an interval-valued fuzzy graphs and also applied for the various types of interval-valued fuzzy graphs and suitable examples have given. We have done some results with examples and relations of split domination number and known parameters in interval-valued fuzzy graph were discussed with the suitable examples. Further, we investigated the relationship of γ_s with other known parameters in Interval-Valued fuzzy graph. Finally we calculated γ_s for some standard interval valued fuzzy graphs.

Competing Interests

Authors have declared that no competing interests exist.

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