



Neural Network Models for Predicting Wellhead Pressure-Flow Rate Relationship for Niger Delta Oil Wells

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Authors' contributions

This work was carried out in collaboration between both authors. Author ANO designed the study, managed the literature searches, analyses the results and prepared the manuscript. Author DA supervised the analyses of the results and important intellectual content in the manuscript. Both authors read and approved the final manuscript.

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ABSTRACT

Some wellhead pressure - flow rate correlations developed for Niger Delta region oil wells are in-house estimation tool by the operating companies in this region. However, the limited available correlations for wellhead pressure - flow rate prediction for Niger Delta oil wells are not generalized. A more robust and adaptable soft computing approach - Artificial Neural Network (ANN) was developed to address the inconsistency using field test data: production flow rate (q), flowing wellhead pressure (P_{wh}), choke size (S), gas-liquid ratio (GLR), flowing temperature (FTHP) and basic sediments and water (BS&W) obtained from 64 oil wells in Niger Delta fields. The developed ANN models were based on Gilbert and modified Gilbert forms of equation for predicting wellhead pressure - flow rate relationship. The results obtained indicate that the developed ANN models resulted in accurate predictions than the empirical correlations. The statistical analysis of the

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developed ANN models predictions with the field test data also resulted in average error, absolute relative error, root mean error and standard deviation of -0.1233, 0.1920, 0.3650 and 0.3621 for Gilbert form and -0.0450, 0.1045, 0.4533 and 0.4498 for modified Gilbert form, respectively. The results also show that the ANN models' prediction resulted in coefficient of determination (R^2) of 0.9653 and 0.9951 for Gilbert and modified Gilbert respectively. The developed ANN models for Gilbert and modified Gilbert predictions are close with coefficient of determination (R^2) of 0.9619. Therefore, the ANN models are superior to the empirical correlations' predictions for wellhead pressure and can be used as a quick-and-robust tool for oilfield prediction of wellhead pressure - flow rate relationship in Niger Delta oil fields.

Keywords: Wellhead pressure; flow rate correlation; artificial neural network (ANN); gilbert form; modified gilbert Form; Niger Delta Region.

1. INTRODUCTION

Wellhead chokes are used in the oil and gas industry to control flow rate, to maintain well allowable, to protect surface equipment, to avert water and gas coning, and to provide the necessary backpressure to reservoir to avoid formation damage from excessive drawdown [1]. Therefore, it controls the surface pressure and production rate from the well. Also, they are selected so that any fluctuations in its line pressure downstream have no effect on the production rate. Available literature to establish multiphase orifice flow correlations can be categorized as analytical models or empirical correlations. Empirical correlation involves using dimensional analysis to select and group the most relevant variables or using field or laboratory data [2]. The later empirical correlation approach is the most widely used method to establish multiphase orifice flow correlations in oil and gas industry. Tangren et al. [3] laid the theoretical framework for gas-liquid two-phase flow through restrictions [4]. Their work was analytical model. Gilbert [5] pioneered work that suggested an empirical correlation for critical flow through choke that predicts liquid flow rates as function of flow wellhead pressure, gas-liquid ratio and surface wellhead choke size. The Gilbert's correlation is expanded in equation 1. In 2007, Ghareeb and Shedid [6] added that Gilbert's correlation is valid for critical flow occurring when the upstream pressure of the choke is at least 70% higher than the downstream pressure or when the ratio of downstream - upstream pressure is equal to 0.588. So far, several researchers have proposed various correlations based on Gilbert's form of correlation. Baxendell [7] developed a revised correlation to Gilbert [5] equation based on incremental data to update the correlation exponents. Ros [8] and Achong [9] presented

another Gilbert's correlation form with modification on the constant and exponents using regression parameters based on data from different oil fields. Similarly, Pilehvari [10] and Beiranvand et al. [11] revised the Gilbert [5] equation with new constant and exponents. In addition, Owolabi et al. [12] and Okon et al. [13] updated the Gilbert's correlation constants for Niger Delta oil wells using data from the region. Table 1 presents the authors' correlation's constants. On the other hand, Beiranvand *et al.* [11] and Khorzoughi et al. [14] introduced additional parameters: basic sediments and water (BS&W) and temperature (T) to modify the Gilbert [5] equation form to predict wellhead pressure-flow rate relationship. This correlation is referred to as modified Gilbert correlation [13], and is expressed in equation 2. Table 2 provides the various authors' correlation's constants for modified Gilbert correlation. In essence, several correlations have been published to describe critical two-phase flow through wellhead chokes, but most of these correlations were based on limited ranges of flow variable [15]. The validity of these developed correlations is limited by the quantity and scope of the data upon which they are based. Regrettably, some correlations are made for in-house estimate by the operating companies in the Niger Delta region. For Niger Delta region oil wells, limited works: Owolabi et al. [12] and Okon et al. [13] are the correlations in the public domain based on data from this region. These developed correlations have accurate wellhead pressure - flow rate predicts over their counterparts developed by other researchers with data from other regions. There are available literatures [16,2] that mentioned the limitation of Gilbert [5] correlation for critical multiphase orifice flow. However, it remains the widely used in the petroleum industry, as it is based on well production parameters. Even the attempt by Beiranvand et al. [11] and Khorzoughi

et al. [14] on this multiphase flow condition still centered on the Gilbert [5] correlation variables with additional two production terms. The reason that makes this empirical approach of multiphase flow parameter estimation frequently use is that the variables are obtained at the surface production conditions. This accounted for its accurate predictions of production rate - wellhead pressure relationship. With this in mind, the modeled neural network models in this work are based on Gilbert [5] and modified Gilbert (2013) correlations' parameters.

The early days of application of artificial neural network (ANN) in petroleum industry dates back to 1989; where it was used in intelligent reservoir simulation interface, drill bit diagnosis and well log interpretation [17]. Recently, its use has gained popularity in petroleum engineering; as several authors have discussed the applications of neural network in oil and gas industry [18]. Therefore, the area of applications of ANN in petroleum industry cannot be overemphasized. Its applications have been in areas where complex parameters are involved in the prediction and analysis of petroleum production systems as well as reservoir responses; during the production of oil and gas reservoirs [19]. Available literatures have provided the application of ANN in areas of multiphase flow regime study, oil PVT (pressure-volume-temperature) properties prediction, well performance prediction, reservoir saturation study, among others. So, for over two decades, this soft computing approach has found its way into the exploration and production (E&P) industry; as predicting tool to study complex system owing to its adaptability and flexibility even with discontinuous systems [20]. In multiphase flow, ANN has been applied in this area and achieved promising results compared to the conventional methods: empirical correlation and mechanistic models. Unfortunately, most of the developed correlations for wellhead pressure - flow rate predictions are very restricted in terms of wide variety of data sets [13]. Also, these developed correlations were based on linear or non-linear multiple regression techniques; an approach

which cannot account for any discontinuity in the system. Recent application of ANN provides an integrated approach for oil and gas well production. With regard to this topic, limited or no work has been reported on the application of ANN technique to predict wellhead pressure - flow rate relationship for Niger Delta region. Therefore, given sufficient actual field data sets, the neural network can be trained to predict wellhead pressure - flow rate much closer to the measured values than those from the established correlations. Thus, this paper evaluates the application of ANN models to predict wellhead pressure - flow rate relationship for Niger Delta oil wells.

$$P_{wh} = \frac{C(GLR)^m q}{S^n} \tag{1}$$

where:

- GLR* = Gas-Liquid Ratio
- q* = Flow Rate
- S* = Choke Size
- P_{wh}* = Flowing Wellhead Pressure
- C, m and n* = Constants

Table 1. Gilbert equation authors' constants

Authors	Correlation constants		
	C	m	n
Gilbert (1954)	10.0	0.546	1.89
Baxendell (1957)	9.56	0.546	1.93
Ros (1960)	17.40	0.50	2.00
Achong (1961)	3.82	0.65	1.88
Pilehvari (1980)	46.67	0.313	2.11
Owolabi et al. (1991)	35.72	0.289	1.83
Beiranvand et al. (2012)	30.49	0.589	2.275
Okon et al. (2015)	5.1474	0.5048	1.7093

$$P_{wh} = \left[\frac{1}{A} \frac{GLR^C q}{S^B \left(1 - \frac{BS\&W}{100}\right)^D \left(T/T_{sc}\right)^E} \right]^{1/F} \tag{2}$$

where:

- T_{sc}* = Surface Temperature (60°F)
- BS&W* = Basic Sediments and Water
- A, B, C, D, E and F* = constants.

Table 2. Modified Gilbert equation authors' constants

Authors	Correlation constants					
	A	B	C	D	E	F
Beiranvand et al. (2012)	0.0382	2.151	0.5154	0.5297	0	1.0
Khorzoughi et al. (2013)	1.0	1.50	0.10	1.0	-0.8	0.5
Okon et al. (2015)	0.0509	1.8134	0.6749	0.2235	0.000029	1.321

2. MATERIALS AND METHODS

2.1 Data Acquisition and ANN Models Development

Data used for this work were collected from sixty four (64) different fields' oil wells in Niger Delta region. The data obtained included flowing wellhead pressure (P_{wh}), production rate (q), choke size (S), gas-liquid ratio (GLR), basic sediments and water (BS&W) and flowing temperature (FTHP). Table 3 presents the statistical description of these input data for the ANN model development. The statistical description of these data was performed using MINITAB 17.1.0 [21]. The neural network model was developed using PYTHIA, developed by Runtime Software [22]. The data were normalized and then fifty percent (50%) of it was used to develop the ANN model for training. The challenge of determining the network structure and optimal numbers of hidden layers and nodes was handled by using evolutionary optimization in the software. The evolutionary optimizer uses evolutionary algorithms for the selection and generation of the fitted neural networks for a given training data set. The fitted ANN models for Gilbert [5] and Modified Gilbert (2013) approaches for wellhead pressure - flow rate relationship prediction are presented in Figs. 1 and 11, respectively. The fitted ANN models indicate that a neural network of three (3) inputs, three levels: 6, 5, 1 neurons (i.e. 12 neurons) and one (1) output predicted the wellhead pressure based on Gilbert [5] approach. While a neural network of five (5) inputs, three levels: 6, 6, 1 neurons (i.e. 13 neurons) and one (1) output predicted the wellhead pressure based on modified Gilbert (2013) approach. The networks were trained at different training level until it was able to predict the given output values (wellhead pressure). Over-training of the networks was avoided by setting the stopping criteria at 500 repetitions, 0.000001 deviations and training rate of 0.8 in the training unit of the PYTHIA software [22]. The aforementioned phenomenon causes the network to memorize the result rather than

generalizes it [17]. Thus, this makes the ANN model to perfectly predict the data similar to training data, but will perform badly when new cases are submitted to the network. After the training phase, the networks become ready for testing and evaluation. To achieve this, the last data set; which the ANN models have not seen during training was used for the validation of the models.

2.2 ANN Models Evaluation

To demonstrate the robustness of the developed ANN models, their predictions were compared with the actual field data and some correlations (as mentioned in this work) for wellhead pressure - flow rate prediction. Average error, absolute error, root mean square, standard deviation and coefficient of determination were used as good indicator of the accuracy of the ANN models. Tables 4 and 5 present the statistical evaluation of the ANN models and some correlations prediction; based on Gilbert [5] and Modified Gilbert (2013), respectively.

3. RESULTS AND DISCUSSION

As earlier alluded to, the modeled ANN models predictions were compared with other empirical correlations prediction to ascertain their performance and accuracy. The selected correlations in the literature include: Gilbert [5], Baxendell [7], Ros [8], Achong [9], Pilehvari [10], Owolabi et al. [12], Beiranvand et al. [11] and Okon et al. [13] for Gilbert equation. While Beiranvand et al. [11], Khorzoughi et al. [14] and Okon et al. [13] were selected for modified Gilbert form of equation.

3.1 Gilbert Approach

For the Gilbert [5] form of equation, Figs. 2 through 10 depict the comparison of the ANN model and the mentioned correlations predictions with the actual field data. Fig. 2 present the ANN model prediction. The Figure indicates that, the ANN model predicted the actual field

Table 3. Description of the input data used for the ANN models formulation

Parameters	Min	Max	Std. dev.	Coef. Var.	Skewness	Kurtosis
Flowing Wellhead Pressure	36	2320	447.20	76.55	1.57	2.92
Choke Size; S	16	76	13.93	39.47	0.82	0.30
Production Rate; q	263	5313	1090.0	67.56	1.19	1.15
Basic Sediments & Water	0	0.880	0.3213	70.64	-0.32	-1.53
Gas-Liquid Ratio; GLR	93	4134	920.0	104.22	2.24	4.93
Flowing Temperature; T	100	150	15.34	12.43	0.18	-1.20

data better than the other empirical correlations. This is noted on the alignment of the field data (field wellhead pressure) and ANN model prediction along the regression line on Fig. 2 compared to other Figures (i.e. Figs. 3 through 10). The outperformance of the ANN model over the empirical correlations is also indicated on its prediction statistical evaluation; as presented in Table 4. The absolute relative error and coefficient of determination (R^2) which are the most important indicator of the accuracy of the ANN model indicates that the model has 0.1920 absolute relative error and 0.9653 coefficient of determination (R^2). Whereas, the empirical correlations resulted in higher absolute relative error and lower coefficient of determination (R^2), as shown in Table 4. This result as presented in Table 4 implies that the ANN model prediction

has 96.53% explained variation with the field wellhead pressure data. While the other empirical correlations have explained variations of 70.14%, 69.98%, 68.97%, 70.67%, 62.38%, 61.85%, 67.11% and 69.41% for Gilbert [5], Baxendell [7], Ros [8], Achong [9], Pilehvari [10], Owolabi et al. [12], Beiranvand et al. [11] and Okon et al. [13], respectively. Also, the less correlation of the actual field data and the empirical correlations prediction is noted on the scattered points on Figs. 3 through 10. Furthermore, the most scattered points were found in Fig. 3, representing Owolabi et al. [12] correlation and Fig. 9, which represent Pilehvari [10] correlation predictions. This observed results indicate their poor performance for this set of data for Niger Delta oil wells' wellhead pressure - flow rate prediction.

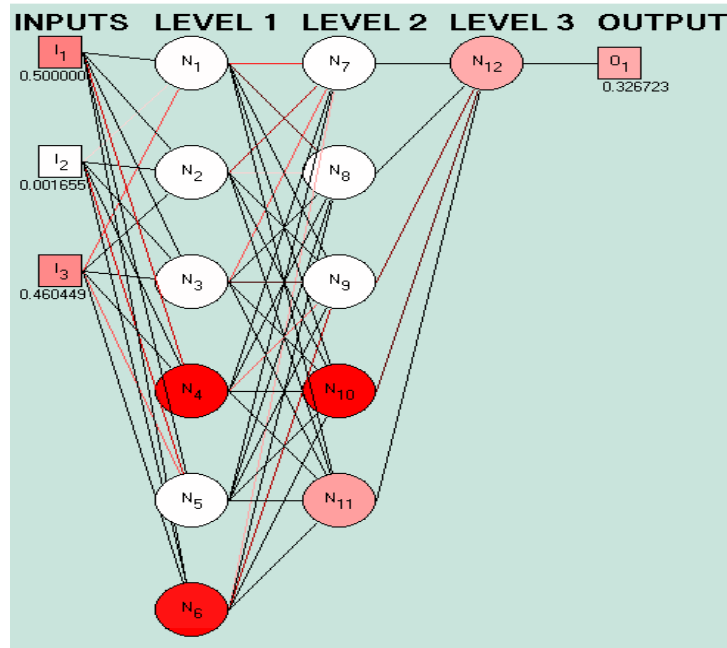


Fig. 1. ANN model for Gilbert approach

Table 4. Gilbert equation statistical analysis

Authors	Average error	Absolute error	Root mean square	Standard deviation	Coefficient of determination
Gilbert (1954)	-0.5604	0.6734	1.4760	1.4645	0.7014
Baxendell (1957)	-0.3000	0.4960	1.1883	1.1790	0.6998
Ros (1960)	-0.3830	0.5409	1.2956	1.2854	0.6897
Achong (1961)	-0.2125	0.4675	1.1022	1.0936	0.7067
Pilehvari (1980)	0.2288	0.4909	0.7380	0.7322	0.6238
Owolabi et al. (1991)	-0.3280	0.5380	1.1762	1.1670	0.6185
Beiranvand et al. (2012)	-0.6890	0.7940	1.7986	1.7845	0.6711
Okon et al. (2015)	-0.1477	0.4430	0.9658	0.9582	0.6941
ANN Model	-0.1233	0.1920	0.3650	0.3621	0.9653

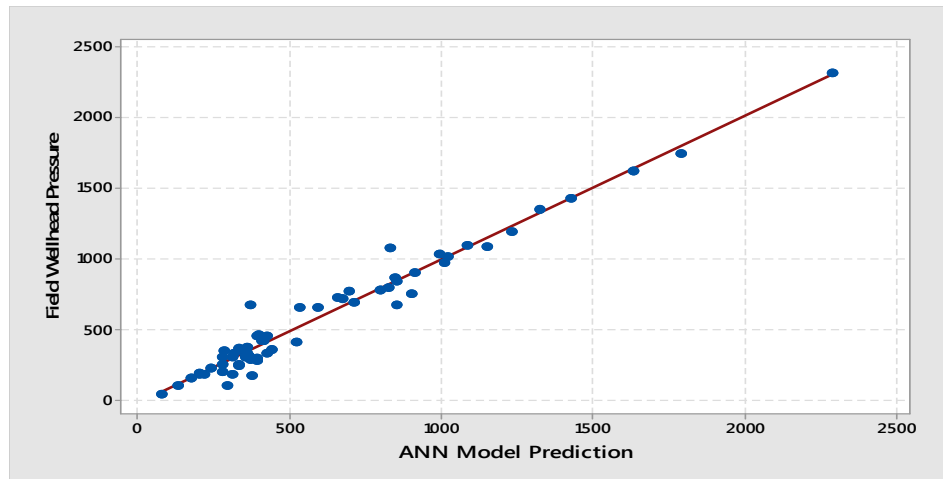


Fig. 2. ANN model prediction (Gilbert approach)

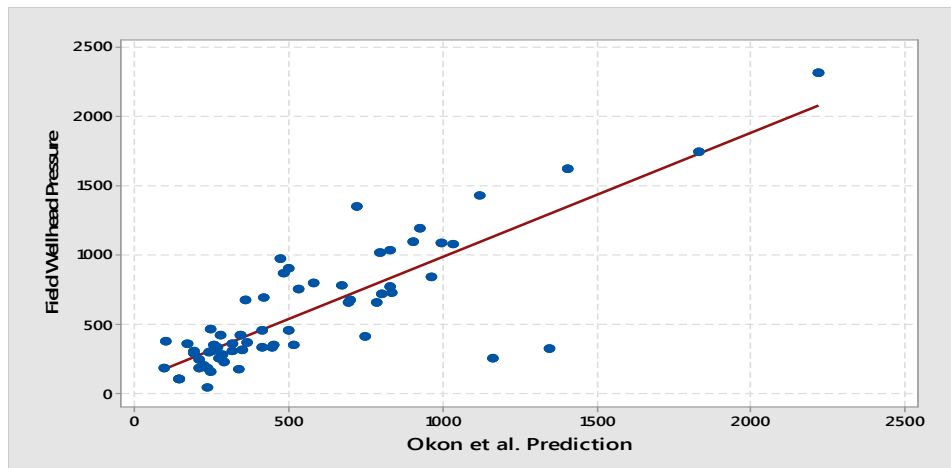


Fig. 3. Okon et al. correlation prediction (Gilbert approach)

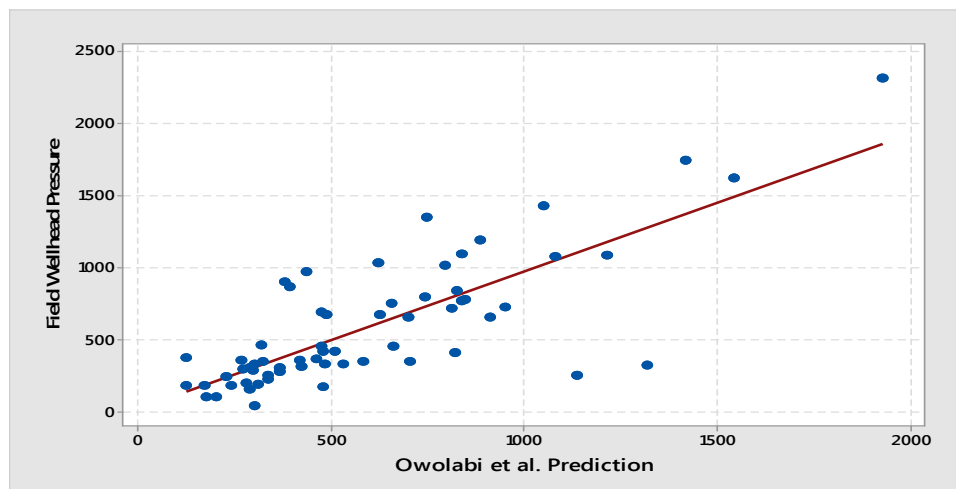


Fig. 4. Owolabi et al. correlation prediction (Gilbert approach)

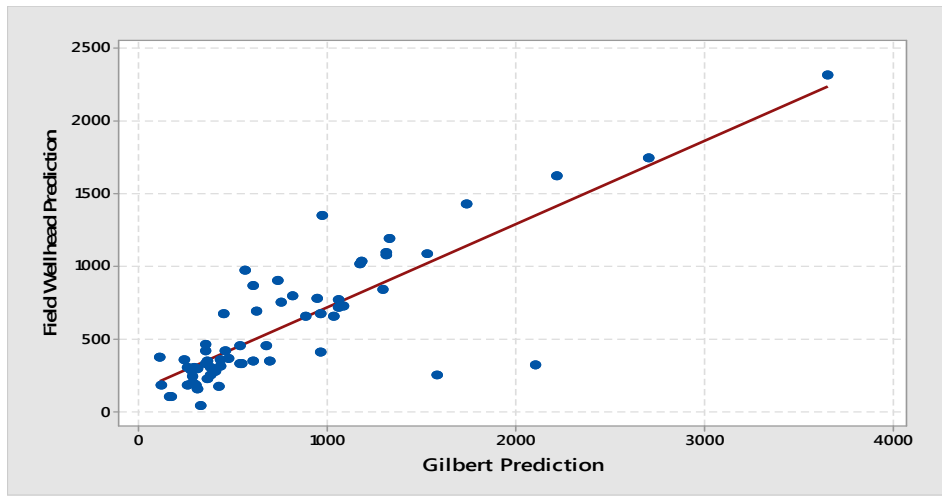


Fig. 5. Gilbert correlation prediction (Gilbert approach)

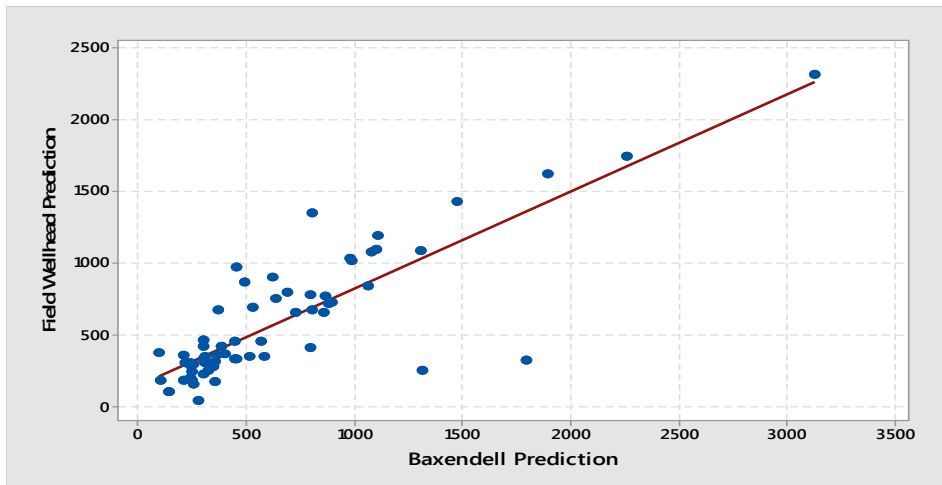


Fig. 6. Baxendell correlation prediction (Gilbert approach)

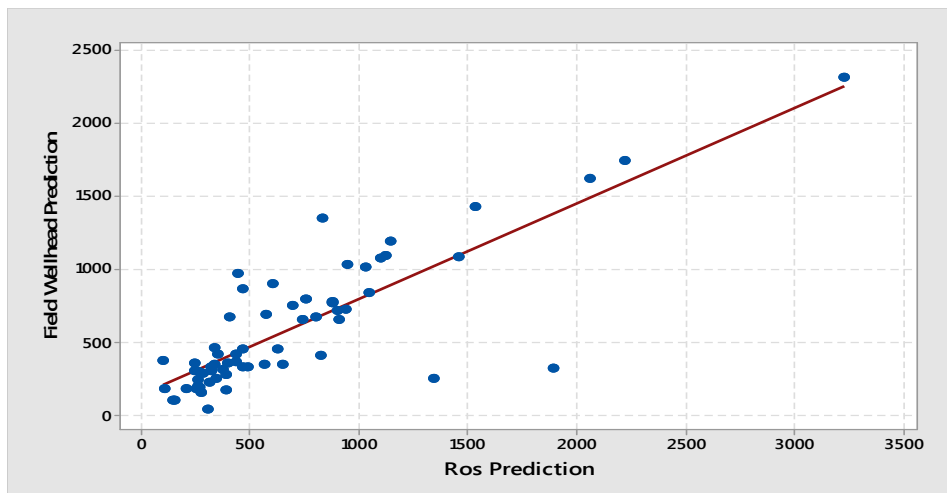


Fig. 7. Ros correlation prediction (Gilbert approach)

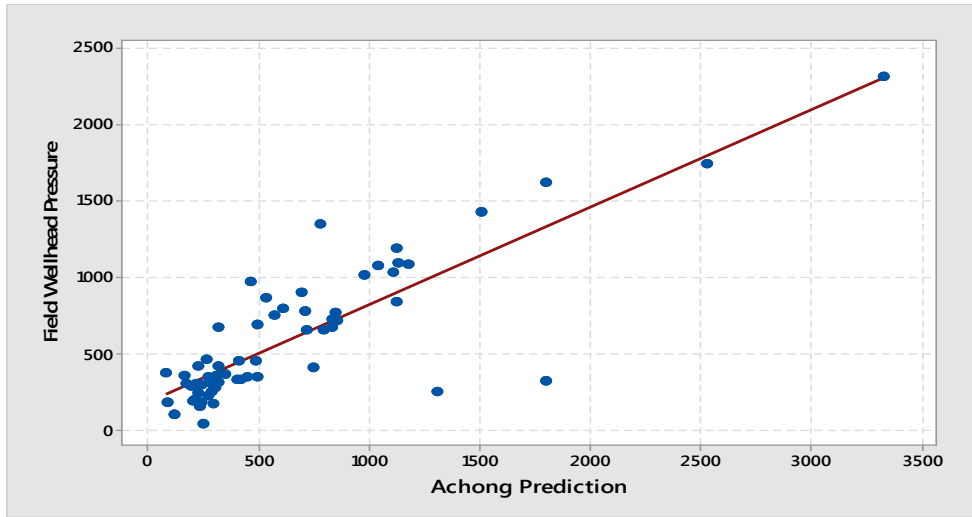


Fig. 8. Achong correlation prediction (Gilbert approach)

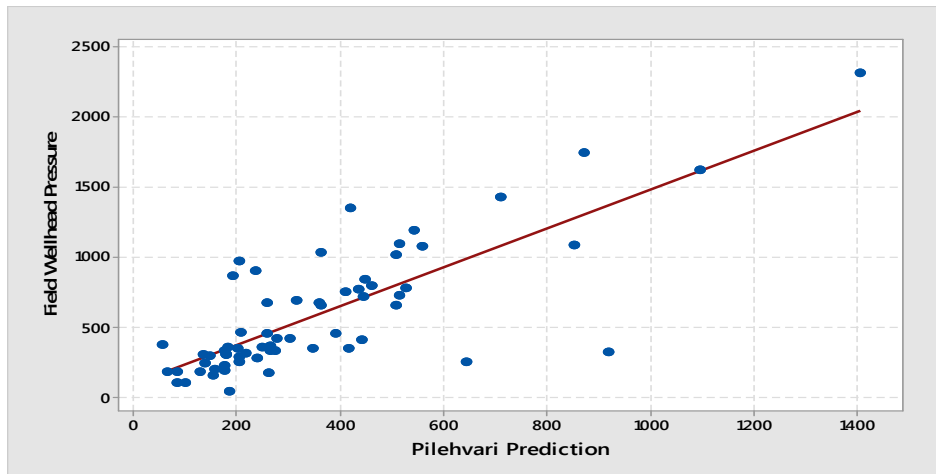


Fig. 9. Pilehvari correlation prediction (Gilbert approach)

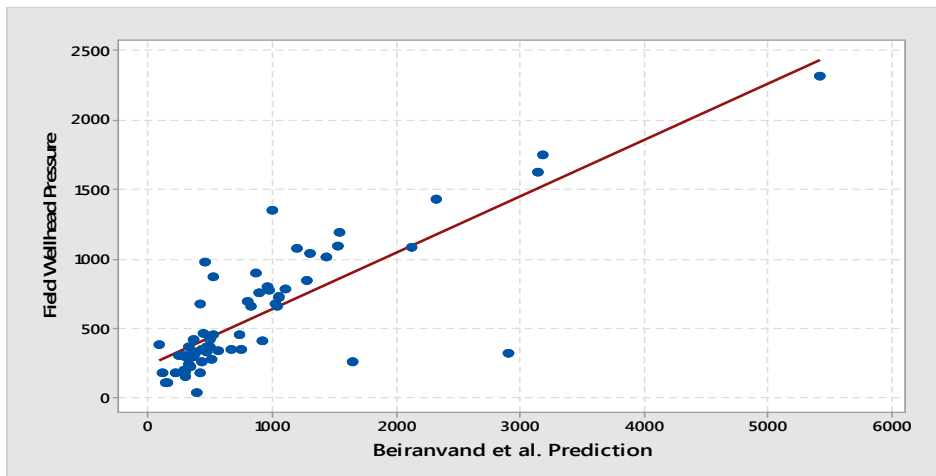


Fig. 10. Beiranvand et al. correlation prediction (Gilbert approach)

3.2 Modified Gilbert Approach

The modified Gilbert approach ANN model and the other empirical correlations predictions are presented in Figs. 12 through 15. Fig. 12 depicts the ANN model prediction compared with the actual field wellhead pressure data. The comparison result indicates a strong correlation between the field data and ANN model prediction. This ascertainment is indicated on the alignment of the scattered points on the regression line on Fig. 12. Also, the coefficient of determination (R^2); as presented in Table 5, indicate a 99.51% explained variation between the ANN model prediction and actual field wellhead pressure data. Additionally, the absolute relative error of the ANN model prediction is 0.1045 compared to the other empirical correlations with 0.5391, 0.9922 and 0.4737 for Beiranvand et al. [10], Khorzoughi et al. [14] and Okon et al. [13], respectively. The

obtained results implies that the ANN model outperform the empirical correlations for wellhead pressure - flow rate prediction. This is observed in the empirical correlations less coefficient of determination (R^2) values, as presented in Table 5. As observed in Figs. 2 and 12, the two developed ANN models closely predicted the field wellhead pressure. In this connection, the developed ANN models wellhead pressure predictions resulted in coefficient of determination (R^2) of 0.9619 as presented in Fig. 16. In all, both ANN models predicted the wellhead pressure - flow rate relationship for Niger Delta oil wells more accurate than the empirical correlations. This shows that the ANN models were well trained. Secondly, the adaptability and flexibility of the ANN model to predict continuous and even discontinuous systems contributed to its outperformance over the empirical correlations.

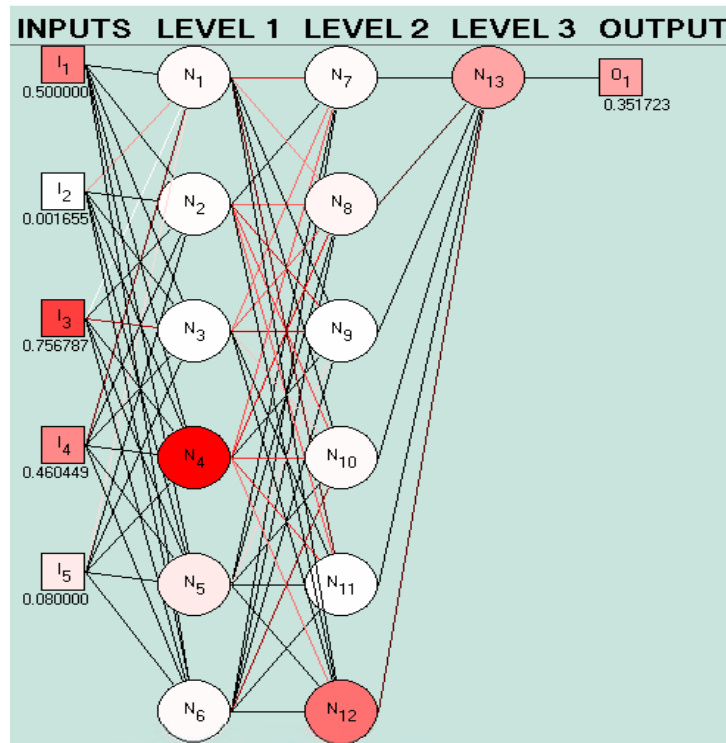


Fig. 11. ANN model for modified Gilbert approach

Table 5. Modified Gilbert approach statistical analysis

Prediction approach	Average error	Absolute error	Root Mean square	Standard deviation	Coefficient of determination
Beiranvand et al. (2012)	-0.3745	0.5391	1.3400	1.3291	0.6784
Khorzoughi et al. (2013)	-0.6711	0.9922	1.5832	1.5708	0.4422
Okon et al. (2015)	-0.2515	0.4737	1.1084	1.0997	0.7185
ANN Model	-0.0450	0.1045	0.4533	0.4498	0.9951

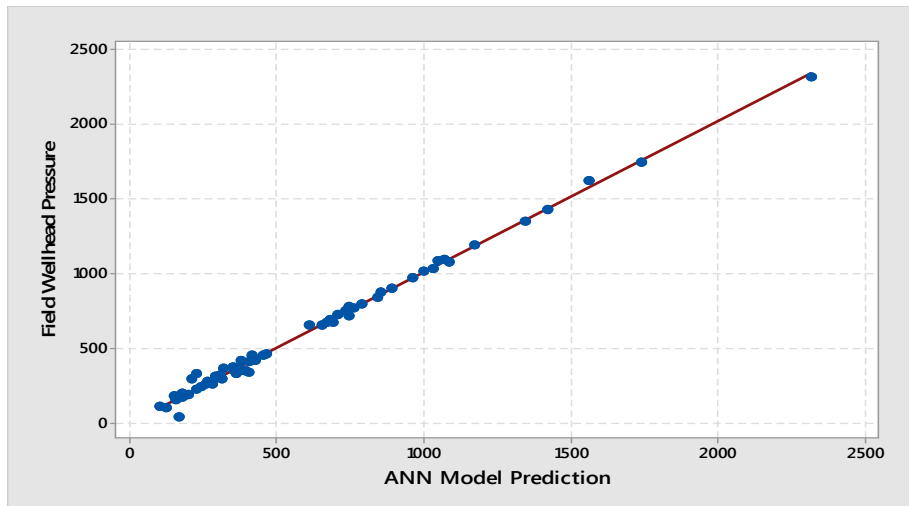


Fig. 12. ANN model prediction (Modified Gilbert approach)

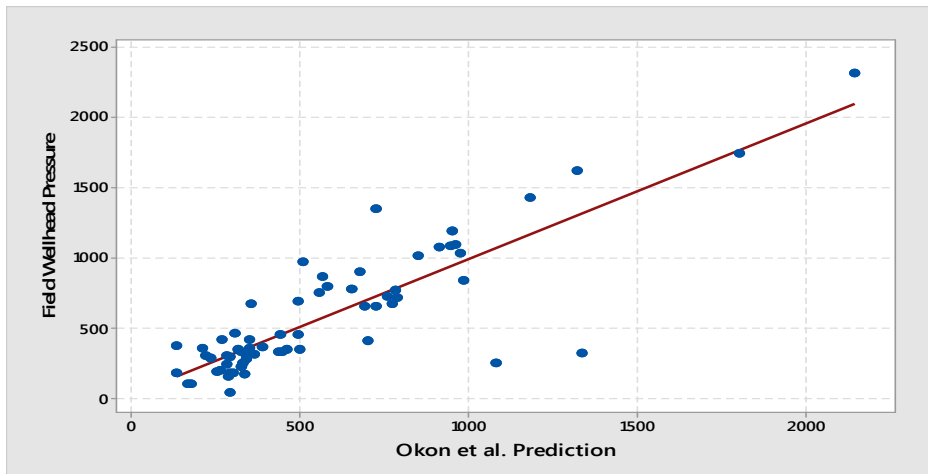


Fig. 13. Okon et al. correlation prediction (Modified Gilbert approach)

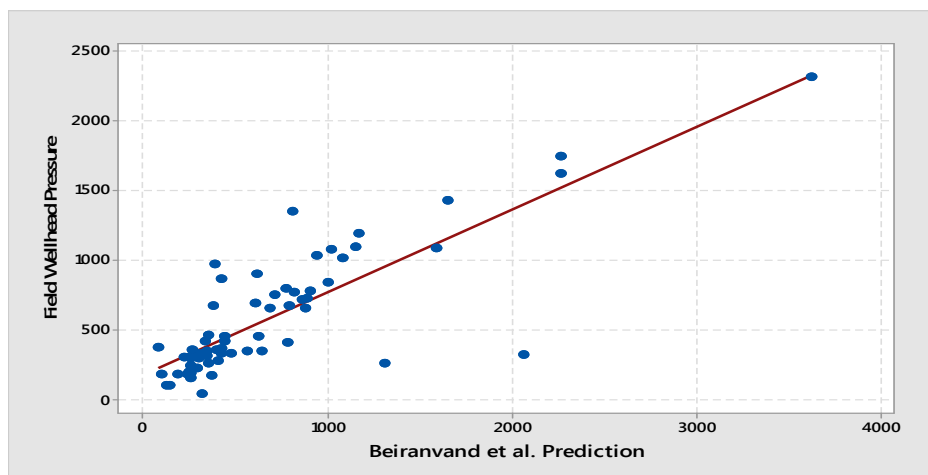


Fig. 14. Beiranvand et al. correlation prediction (Modified Gilbert approach)

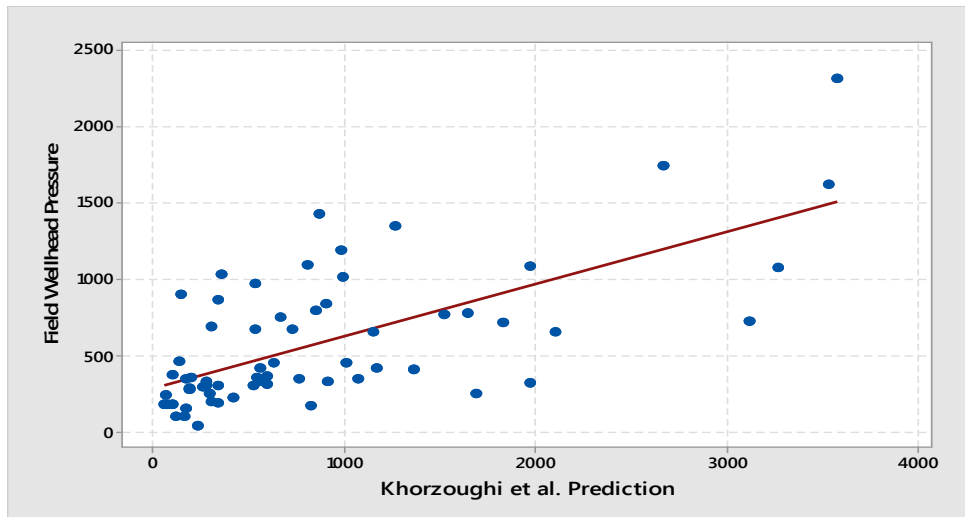


Fig. 15. Khorzoughi et al. correlation prediction (Modified Gilbert approach)

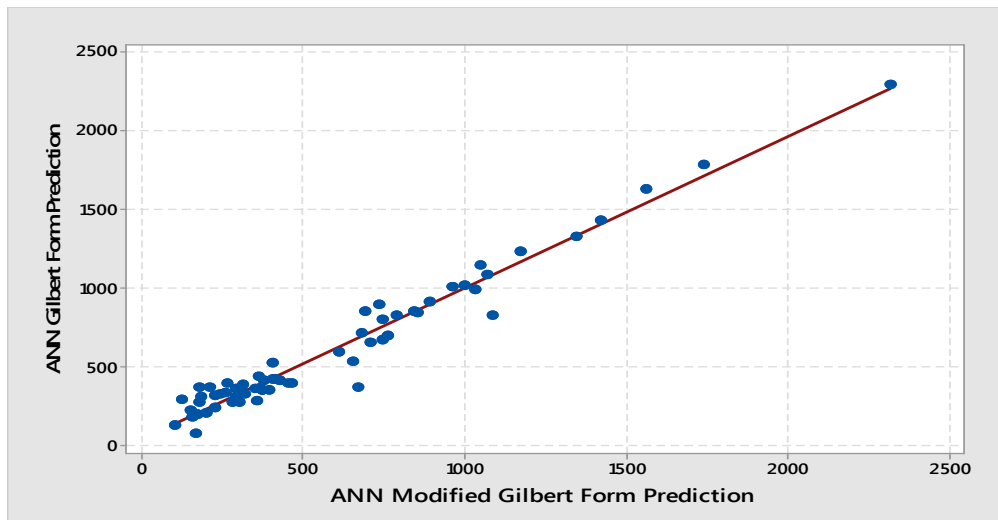


Fig. 16. Comparison of the ANN models: Gilbert and Modified Gilbert predictions

4. CONCLUSION

Most of the developed correlations to predict wellhead pressure - flow rate relationship are based on data obtained from field(s) outside Niger Delta region. Their predictions for Niger Delta oil wells are lower or higher than the expected field values when applied to the aforementioned region. On the other hand, some developed wellhead pressure - flow rate correlations based on Niger Delta field data are in-house equations by the operating companies in the Niger Delta region. Regrettably, these correlations are controlled as proprieties for in-house estimation by the operating companies.

Although there are some correlations in the literature for wellhead pressure - flow rate predictions based on data from Niger Delta region, but these correlations lacks flexibility and adaptability due to its variability on data range. A more robust and adaptable soft computing approach - artificial neural network (ANN) models were developed. It resulted in favourable prediction for wellhead pressure - flow rate based on field test data obtained from sixty four (64) oil wells in Niger Delta region. The ANN prediction when compared to some existing correlations indicates it outperform them, based on the following results:

1. The developed ANN models predict the field wellhead pressure with absolute relative error and coefficient of determination (R^2) of 0.1920 and 0.9653 for Gilbert equation form and 0.1045 and 0.9951 for modified Gilbert equation form
2. The ANN models' standard deviation for Gilbert form and modified Gilbert form are 0.3621 and 0.4498 respectively
3. The developed ANN models based on Gilbert form and modified Gilbert form approaches are comparable with coefficient of determination (R^2) of 0.9619.
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In conclusion, the developed ANN models' predictions compared to the empirical correlations depict high precision from statistical analysis. Therefore, the developed ANN models can serve as a practical and robust tool for oilfield prediction of wellhead pressure - flow rate relationship in Niger Delta region oil wells.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDIX

The equations used for statistical analysis of the ANN model and empirical correlations' predictions:

1. Average error:

$$E_{avg} = \frac{1}{N} \sum_{i=1}^N \frac{(P_{wh})_{field_i} - (P_{wh})_{model_i}}{(P_{wh})_{field_i}}$$

2. Absolute error:

$$E_{abs} = \frac{1}{N} \sum_{i=1}^N \left| \frac{(P_{wh})_{field_i} - (P_{wh})_{model_i}}{(P_{wh})_{field_i}} \right|$$

3. Root mean square error:

$$E_{rms} = \frac{1}{N} \sum_{i=1}^N \left(\frac{(P_{wh})_{field_i} - (P_{wh})_{model_i}}{(P_{wh})_{field_i}} \right)^2$$

4. Standard deviation:

$$S_{dev} = \frac{1}{N} \sqrt{N \sum_{i=1}^N \left(\frac{(P_{wh})_{field_i} - (P_{wh})_{model_i}}{(P_{wh})_{field_i}} \right)^2 - \sum_{i=1}^N \left(\frac{(P_{wh})_{field_i} - (P_{wh})_{model_i}}{(P_{wh})_{field_i}} \right)^2}$$

5. Coefficient of determination:

$$R^2 = \left(\frac{\frac{1}{N} \sum ((P_{wh})_{field_i} - (\overline{P_{wh}})_{field}) ((P_{wh})_{model_i} - (\overline{P_{wh}})_{model})}{\sqrt{\left[\frac{1}{N} \sum ((P_{wh})_{field_i} - (\overline{P_{wh}})_{field})^2 \right] \left[\frac{1}{N} \sum ((P_{wh})_{model_i} - (\overline{P_{wh}})_{model})^2 \right]}} \right)^2$$

where:

$P_{wh\ field_i}$ = Field Wellhead Pressure

$P_{wh\ model_i}$ = Model Predicted Wellhead Pressure

$(\overline{P_{wh}})_{field}$ = Average Field Wellhead Pressure

$(\overline{P_{wh}})_{model}$ = Average Model Predicted Wellhead Pressure

N = Number of Wellhead Pressures

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