British Journal of Mathematics & Computer Science
18(6): 1-9, 2016, Article no.BJMCS.29055
ISSN: 2231-0851

SCIENCEDOMAIN *international*
www.sciencedomain.org

Improvement of Threshold Function Based on Wavelet De-noising

Yi Lingzhi[1], Xiao Weihong[1*], Yu Wenxin[2] and Wang Genping[3]

[1]Hunan Province Cooperative Innovation Center for Wind Power Equipment and Energy Conversion, College of Information Engineering, Xiangtan University, Hunan, Xiangtan, 411105, P.R. China.
[2]Hunan University of Science and Technology, Hunan, Xiangtan, 411201, P.R. China.
[3]Shenzhen Polytechnic, Shenzhen, 518000, Guangdong, P.R. China.

Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/BJMCS/2016/29055
Editor(s):
(1) Victor Carvalho, Polytechnic Institute of Cávado and Ave, Portuguese Catholic University and Lusiada University, Portugal.
Reviewers:
(1) S. K. Srivatsa, PITAM, Chennai, India.
(2) Zhi Shang, Louisiana State University, USA.
(3) Irshad Ullah, Government of KPK, Pakistan.
(4) Ferda Ernawan, Universiti Malaysia Pahang, Malaysia.
Complete Peer review History: http://www.sciencedomain.org/review-history/16343

Original Research Article

Received: 20th August 2016
Accepted: 17th September 2016
Published: 27th September 2016

Abstract

Hard threshold function is discontinuity in the threshold point, and soft threshold function have a constant error between the estimated coefficient of wavelet and the original coefficient of wavelet. To solve this problem, an improved threshold function is presented, which is continuous and high-order-differential in the threshold point. The experimental results show that the de-noising effect of this function is better than the soft threshold function, the hard threshold function and the modular square function.

Keywords: Wavelet threshold; image de-noising; improved methods; threshold function.

1 Introduction

Image signal is often mingled with noise in transmission process, which has impact to the subsequent signal processing in a way, so image signal de-noising is very meaningful. Common de-noising methods are mathematical morphological method, adaptive smoothing, random method, filter method, wavelet de-noising

**Corresponding author: E-mail: 201431111830@smail.xtu.edu.cn, 475939526@qq.com;*

method. Because wavelet has characteristics of low entropy, multi-resolution, decorrelation and flexibility of the selection, wavelet de-noising has become the most widely used method in the application [1,2]. Wavelet de-noising method is generally divided into three categories: the first method is correlation de-noising method [3], that is, to calculate the correlation of the wavelet coefficients between adjacent scales, then eliminate unqualified coefficients, finally reconstruct the signal; The second method is wavelet modulus maxima de-noising method [4]. The propagation characteristics of signal and noise are different in each scale of wavelet transform, so as long as remove the modular maximum value from noise, and keep the corresponding modulus maxima of signal, and then reconstruct the wavelet coefficients, we can recover signal and remove noise. The third category is wavelet threshold method [5]. The wavelet coefficients of signal contain important information and its amplitude is large, but the number of wavelet coefficients is less. Instead, the wavelet coefficients of noise are uniformly distributed and its amplitude is low, but the number of wavelet coefficients is major. So if the absolute value of the smaller coefficient is set to zero, then retain the absolute value of larger coefficient, and reconstruct signal to de-noising.

Nowadays, there are many traditional methods in the field of image de-noising. Traditional methods can filter noise, but at the same time they make the image details blurred. Therefor, it is significant to further improve the existing method. Different algorithms have different mathematical basis, and the effect of image de-noising is also different. By studying the different algorithms to improve the de-noising effect is very meaningful too. Based on the above situation, this paper mainly studied the wavelet threshold de-noising method.

The main structures of this article are as follows: the principle of wavelet threshold de-noising will be introduced in second section. In third section, the method of hard threshold and soft threshold method will be presented, and their disadvantages will be discussed. In order to solve the problems existing in traditional threshold function, an improved threshold function is presented. MATLAB simulation experiment is used to prove the effectiveness and superiority of improved function in fourth section. The fifth section is the summary.

2 Wavelet Threshold De-noising

Wavelet threshold method is a typical noise suppression method based on non-parametric model. After wavelet decomposition, the energy of image itself is mainly distributed in the low resolution scale function, and the energy of noise is still distributed evenly in the low resolution scale function and all the wavelet coefficients. In the transformational domain, the spatial correlation of image is reduced and energy is more concentrated, and energy distribution of noise is constant. Based on this feature, wavelet de-noising technology retains or shrinks large wavelet coefficients and remove small wavelet coefficients, which called the wavelet threshold method.

Signal based on wavelet threshold de-noising process can be divided into the following steps [6-8]:

Step1. Select the appropriate wavelet basis and wavelet decomposition layers, the calculation of wavelet decomposition coefficient with noise signal.

Step2. Select the appropriate threshold, for the high frequency coefficient threshold processing.

Step3. According to the processed wavelet coefficients reconstruct signal.

The specific processes are as shown in Fig. 1.

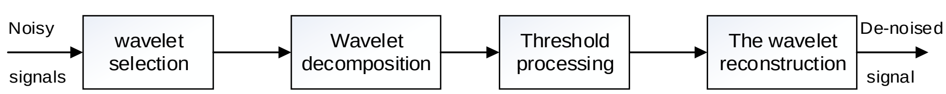

Fig. 1. Process of signal threshold

2.1 Traditional threshold function

It is one of the most important problems in wavelet de-noising algorithm that how to select threshold function, which is related to the quality of signal de-noising. The traditional threshold de-noising method can be divided into the hard-threshold de-noising method and the soft-threshold de-noising method. The expression is shown as following.

The expression of hard-threshold function is

$$\hat{w}_{j,k} = \begin{cases} w_{j,k} & \left|w_{j,k}\right| \geq \lambda \\ 0 & \left|w_{j,k}\right| < \lambda \end{cases} \tag{2.1}$$

The expression of soft-threshold function is

$$\hat{w}_{j,k} = \begin{cases} \operatorname{sgn}(w_{j,k})[\left|\hat{w}_{j,k}\right| - \lambda] & \left|\hat{w}_{j,k}\right| \geq \lambda \\ 0 & \left|\hat{w}_{j,k}\right| < \lambda \end{cases} \tag{2.2}$$

Where $w_{j,k}$ stands for the wavelet coefficients, $\hat{w}_{j,k}$ stands for the wavelet coefficients after treatment. λ is the threshold.

Although hard and soft threshold de-noising method proposed has been widely used in the actual process of image de-noising, but it is still insufficient [9-10]. In the hard threshold method, the wavelet coefficients processed by the threshold are discontinuous at the two points λ and $-\lambda$, therefore, which may lead to the reconstructed signal prone to Pseudo-Gibbs phenomenon and a certain degree of oscillation will directly lead to the increasing error. In soft threshold function, the continuity of estimated wavelet coefficients is good on the whole and doesn't produce additional shocks, but there is a constant deviation between $\hat{w}_{j,k}$ and $w_{j,k}$ when $\left|\hat{w}_{j,k}\right| \geq \lambda$. This will cause a deviation between the reconstructed signal and the real signal. In addition, the derivative of soft threshold function is discontinuous. But in actual applications, it is often processed by the first derivative and even higher order derivative, so soft threshold function has some limitations.

2.2 Threshold selection

The selection of wavelet threshold is related to the effect of de-noising. If the selection of threshold value is too low, the de-noising effect is not ideal; On the contrary, if the selection of threshold is too higher, some important edge detail characteristics of image will be filtered to remove while removing noise and the image is too smooth [11].

The expression of Traditional threshold selection function (see [12]) is

$$\lambda = \sigma\sqrt{2\log N} \tag{2.3}$$

Where σ is the standard deviation of the noise, and N stands for image size.

3 The Improved-threshold Function

Soft and hard threshold de-noising methods, although it has been widely applied and achieved well results in practice, but these two kinds of method itself have some potential disadvantages. For example, high-order-differential does not exist in the soft threshold function at the threshold point, the hard threshold function is not continuous at the threshold point, and these will have a certain impact on the de-noising effect. In order to overcome the shortcomings of the two methods above, an improved method based on them is proposed. The function can be adjusted by 3 parameters, which can effectively reduce the constant error between this signal and the original signal. It not only inherits the advantages of the soft and hard threshold, but also overcomes their shortcomings, and achieves the desired purpose.

Its function is as followed:

$$\hat{w}_{j,k} = \begin{cases} \mathrm{sgn}(w_{j,k})[a(\left|w_{j,k}\right| - \lambda + \frac{\lambda}{2\beta+1} + (1-a)\sqrt{{w_{j,k}}^2 - \lambda^2}] + b\sin(\frac{\pi}{2\lambda}w_{j,k}) & \left|w_{j,k}\right| \geq \lambda \\ 0 & \left|w_{j,k}\right| < \lambda \end{cases} \tag{3.1}$$

Where, a、 β and b are variable parameters, the function is continuous at the threshold, and the high order derivative of the function is continuous when $\left|w_{j.k}\right| > \lambda$. $\hat{w}_{j,k}$ will be more and more close to $w_{j,k}$ when $\left|w_{j.k}\right|$ continues to increase.

Proof：

(1) If $w_{j,k} = \lambda$, we get

$$\hat{w}_{j,k} = \mathrm{sgn}(w_{j,k})[a(\left|w_{j,k}\right| - \lambda + \frac{\lambda}{2\beta+1}) + (1-a)\sqrt{{w_{j,k}}^2 - \lambda^2}] + b\sin(\frac{\pi}{2\lambda}w_{j,k}) = \frac{a\lambda}{2\beta+1} + b$$

$$= \hat{w}_{j,k}\Big|_{w_{j,k}\to\lambda} = \lim_{w_{j,k}\to\lambda}[\frac{a(w_{j,k})^{2\beta+1}}{(2\beta+1)\lambda^{2\beta}} + b\sin(\frac{\pi}{2\lambda}w_{j,k})] = \frac{a\lambda}{2\beta+1} + b$$

This function is continuous at λ, and the same can be proved at $-\lambda$, that is, the function is continuous at the threshold.

(2) If $w_{j,k} > \lambda$, we have

$$(\hat{w}_{j,k})' = \{\mathrm{sgn}(w_{j,k})[a(w_{j,k} - \lambda + \frac{\lambda}{2\beta+1}) + (1-a)\sqrt{{w_{j,k}}^2 - \lambda^2}] + b\sin(\frac{\pi}{2\lambda}w_{j,k})\}'$$

$$= a + \frac{(1-a)w_{j,k}}{\sqrt{{w_{j,k}}^2 - \lambda^2}} + \frac{b\pi}{2\lambda}\cos(\frac{\pi}{2\lambda}w_{j,k})$$

The derivative of the function is continuous when $w_{j,k} > \lambda$, and the same can be proved when $w_{j,k} < -\lambda$, that is, the derivative of the function is continuous when $\left|w_{j.k}\right| > \lambda$. Obviously the high order derivative of the function is continuous when $\left|w_{j.k}\right| > \lambda$.

(3)

$$\lim_{w_{j,k}\to+\infty} = \frac{\hat{w}_{j,k}}{w_{j,k}} = \lim_{w_{j,k}\to+\infty} \frac{a(w_{j,k}-\lambda+\frac{\lambda}{2\beta+1})+(1-a)\sqrt{w_{j,k}{}^2-\lambda^2}+b\sin(\frac{\pi}{2\lambda}w_{j,k})}{w_{j,k}}$$

$$= \lim_{w_{j,k}\to+\infty}[a+(1-a)\frac{\sqrt{w_{j,k}{}^2-\lambda^2}}{w_{j,k}}+\frac{b}{w_{j,k}}\sin(\frac{\pi}{2\lambda}w_{j,k})]$$

$$= \lim_{w_{j,k}\to+\infty}[a+(1-a)\sqrt{1-\frac{\lambda^2}{w_{j,k}{}^2}}+0]=1$$

$\hat{w}_{j,k}$ is equal to $w_{j,k}$ when $|w_{j.k}|$ tend to ∞ .

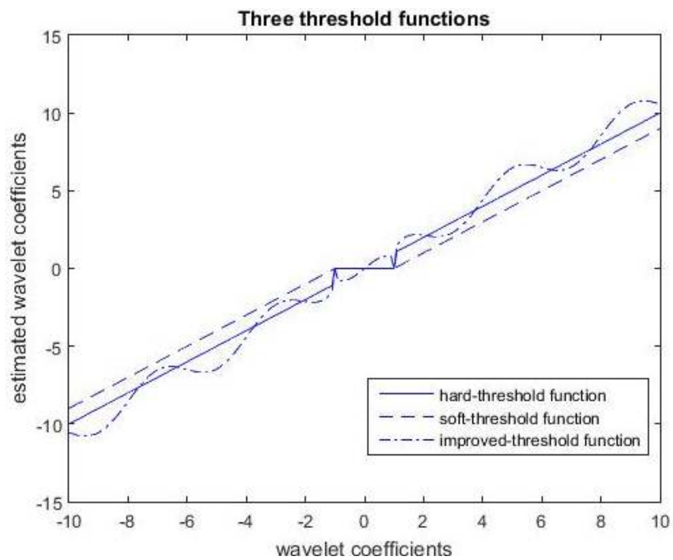

Fig. 2. Three kinds of threshold function curves

Fig. 2 is the hard threshold function, soft threshold function and the improved threshold function of the curve. Where, $\lambda=1, \beta=1, a=2, b=1$.

As we can see from Fig. 2, hard-threshold function does not have continuity in the threshold point, high-order-differential doesn't exist in soft-threshold function at the threshold point, but the improved threshold function is continuous and high-order-differential at the threshold point, which overcomes the shortcomings of the traditional threshold function, and achieves the desired results.

4 Experimental Results

In order to show the new threshold function de-noising effect, this article used the hard threshold function, soft threshold function , modular square function (see [13]) and the improved threshold function based on Matlab to remove the noise of images of devil, Catherine and wflower which contained 0.1 Gaussian noise by simulation experiments. We all choose "rbio6.8" wavelet to 2 layer wavelet decomposition that contain noise image, and $\lambda=\sqrt{2\log N}, \beta=1$.

Experimental results are shown in Figs. 3, 4, 5. The data in experimental simulation are shown in Table 1.

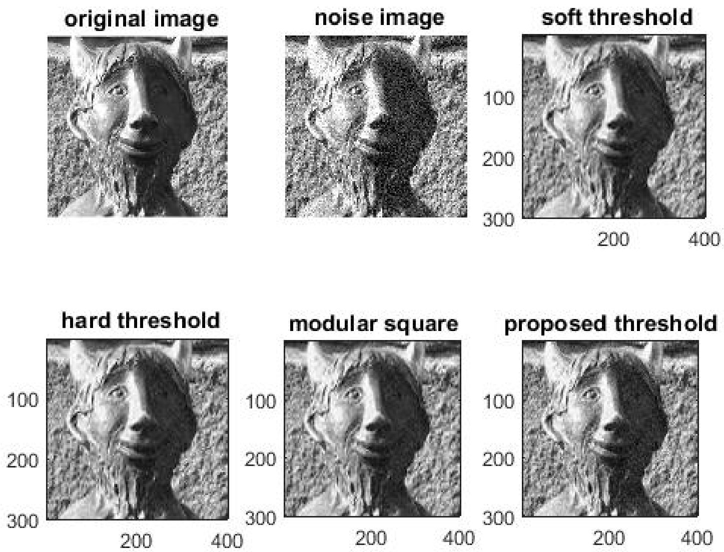

Fig. 3. Devil de-noising effect chart (a=-0.8, β=1, b=20)

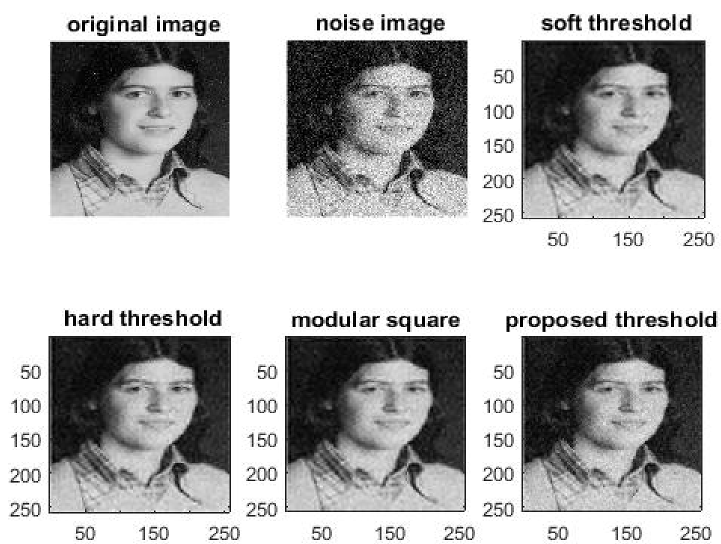

Fig. 4. Catherine de-noising effect chart (a=-0.8, β=1, b=7)

Table 1. Comparison of four kinds of threshold function de-noising

Date	Signal	Soft threshold	Hard threshold	Modular square	Proposed threshold
MSE	Devil	402.2432	383.8176	385.3568	243.7073
	Catherine	195.6408	191.4148	190.9599	155.1403
	Wflower	482.2878	445.7197	453.2064	270.6761
PSNR	Devil	22.0859	22.2896	22.2722	24.2621
	Catherine	25.2162	25.3110	25.3214	26.2236
	Wflower	21.2977	21.6402	21.5678	23.8063

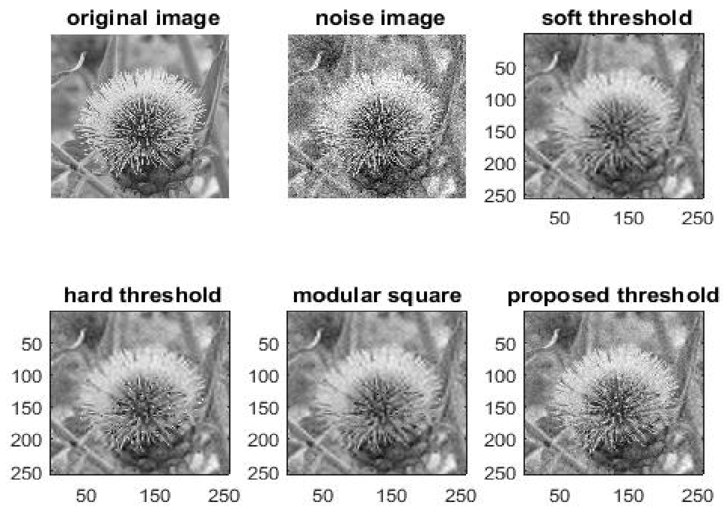

Fig. 5. Wflower de-noising effect chart (a=-1.8, β=1, b=22)

The devil signal β value de-noising effect comparison are show in Table 2.

Table 2. β comparison of the parameters

β	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**
MSE	456.2730	243.7073	308.3281	280.6986	308.4620	296.9577
PSNR	21.5386	24.2621	23.2336	23.6884	23.2388	23.4039

The devil signal a value de-noising effect comparison are show in Table 3.

Table 3. a comparison of the parameters

a	**-1.2**	**-1**	**-0.8**	**-0.6**	**-0.4**	**-0.2**
MSE	245.6282	245.3365	243.5469	249.7632	250.0588	251.7874
PSNR	22.2280	24.2332	24.2650	24.1555	24.1504	24.1205

The devil signal b value de-noising effect comparison are show in Table 4.

Table 4. b comparison of the parameters

b	**14**	**17**	**20**	**23**	**26**	**29**
MSE	253.6766	246.7555	244.3815	245.6854	247.4836	248.0187
PSNR	24.0880	24.2081	24.2501	24.2270	24.1953	24.1860

As we can see from Figs. 3-5, the new threshold function image on the visual effect is better than soft threshold function, hard threshold function and modular square function. This function preserves more details of original images too. Table 1 quantitatively describes the superiority. The new threshold function has been improved by 8.97% compared to the other functions when de-noising signal is devil. The new threshold function has been improved by 3.84% compared to the other functions when de-noising signal is Catherine. And the new threshold function has been improved by 10.5% compared to the other functions when de-noising signal is wflower. It can be very intuitive to get that the PSNR is the highest, and the MSE is the least.

Through Table 2, we can see that the PSNR is the highest, and the MSE is the least when β is equal to 1. That is to say, the optimal values of the parameters of the devil signal is $\beta = 1$. In the same way ,we can know that the optimal values of the other two parameters of the devil signal is $a = -0.8, b = 20$. In a word, we can determine the optimal values of the three parameters of the devil signal is $a = -0.8, \beta = 1, b = 20$.

In summary, new threshold function is not only effectively removed Gaussian noise, but also is better to keep the image edge information, improving the peak signal to noise ratio of the image.

5 Conclusion

A novel wavelet threshold de-noising function proposed in this article which can adjust its parameter. Through mathematical analysis, we demonstrate the superiority of this function. And the experimental results show that the de-noising effect of this function is better than the soft threshold function, the hard threshold function and the modular square function. Therefore, the improved method is a practical and effective method to remove the noise of the images.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (61572416), Shenzhen significant strategy layout project (JCYJ20160429112213821), and Hunan province Natural science Zhuzhou United foundation (2016JJ5033).

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Shiwani Saini, Lillie Dewan. Application of discrete wavelet transform for analysis of genomic sequences of *Mycobacterium tuberculosis* [J]. Springer Plus. 2016;5(1):1-15.

[2] Wang Huan. Research on multi-sensor remote sensing image fusion based on wavelet transform [D]. Xiangtan University; 2009.

[3] Song Jingpan. Research on image denoising based on wavelet analysis [J]. China New Telecommunications. 2015;03:115.

[4] Wang Zongliang. Signal processing and performance improvement for distributed fiber raman temperature sensor [D]. Shandong University; 2015.

[5] Donoho DL, Johnstone IM. Adapting to unknown smoothness via wavelet shrinkage [J]. American Statical Assoc. 1995,90(432):1200-1224.

[6] Xin Yuanchao, Zhang Junan, Fang Zhou. Improved adaptive new wavelet threshold denoising algorithm [J]. Computer & Digital Engineering. 2016;06:1151-1153-1175.

[7] Zhu Weihua, An Wei, You Lihua, Wu Jingjing. Wavelet threshold denoising algorithm based on improved wavelet threshold function [J]. Computer Systems & Applications. 2016;06:191-195.

[8] He Lingli, Wang Yufeng, He Wenjing. Application of modified threshold method based on wavelet transform in ECG signal de-noising [J]. Biomedical Engineering and Clinical Medicine. 2016;02:127-130.

[9] Li Xiaofei, Qiu Xiaohui. An improved soft-threshold image denoising algorithm based on wavelet transform [J]. Computer Technology and Development,2016,05:76-78.

[10] Peng Jishen, Zhang Zhichao, Han Yanchun. A denoising algorithm based on a new threshold function appling adaptive learning algorithm [J]. Microelectronics & Computer. 2016;04:81-85.

[11] Mingyue Zhou, Wenlong Jiang. Image hybrid denoise algorithm based on wavelet transform. Journal of Chang Chun University of Technology (Natural Science Edition). 2008;29(2):162-164.

[12] Ju Zhang, Chen Wang, Yun Cheng. Despeckling for medical ultrasound images based on wavelet and bilateral filter. Journal oflmage and Graphics. 20l4;19(l):126-132.

[13] Zhao Ruizhen, Song Guoxinag, Wang Hong. Better threshold estimation of wavelet coefficient for improving denoising [J]. Journal of Northwestern Polytechnical University. 2001;04:625-628.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://sciencedomain.org/review-history/16343