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An Efficient Seven-point Hybrid Block Method for the Direct Solution of $y'' = f(x, y, y')$

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Abstract

A Self starting Stomer Cowell Hybrid Block Method of uniform Order 8 for Direct solution of General Second Order Initial Value Problems of Ordinary Differential Equations is presented in this paper. The method produces simultaneous approximation of the solution in a Block of Seven points at $x = x_{n+j}\ j = 1, 2, 3, 4, \frac{13}{3}, \frac{9}{2}$ and $\frac{14}{3}$ at once. The Block Analysis of this method shows that it is Zero stable and convergent. Numerical experiments show there superiority over the existing method.

Keywords: A Seven points, Hybrid Block Method, Multi-step, General Second Order, Initial value Problems.

1 Introduction

Efforts are directed towards improving the Uniform Order 6 Hybrid Block Method in [1] to a New Uniform Order 8 Hybrid Block Method for the solution of General Second Order Ordinary Differential Equations of the form

$$y'' = f(x, y, y')\ y(0) = \alpha\ , y'(0) = \beta \qquad (1)$$

In the past, [2,3] have developed methods of solving equation (1). In most of these Authors, the original problem is normally reduced to a system of first order ODEs and the Numerical methods meant for first order is now applied on them. In their approach, Numerical instability normally set in because the reduced form may not pose as the original problems.

Also [4,5], developed method for (1) in Predictor-Corrector mode of implementations which is cumbersome and tedious because of the off-grid points present in the method.

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The aim of this paper is to generate Seven points Block method at $k = 5$ and obtained a uniform Order 8 schemes to form the Block without requiring a starting value and produces better results than [6,7,8]. This new approach eliminates the tediousness of Predictor-Corrector mode of implementation used in [2-5]. The method is cost effectiveness because seven different points are obtained at once which speed up the Computation processes and it is numerically stable.

1.1 Definition 1

A Linear multi-step Method of the form

$$\boldsymbol{y}(x) = \sum_{j=0}^{t+m-1} \alpha_j (x) y_{n+j} + h^2 \sum_{j=0}^{t+m-1} (\beta_j (x) f_{n+j} + h^2 \beta_u (x) f_{n+u} + h^2 \beta_v (x) f_{n+v}) \quad (2)$$

is said to both Stomer cowell and Hybrid method of two off-grid points if and only if

$$\sum_{j=0}^{t+m-1} \alpha_j = 0 \quad and\ x\epsilon\ (x_n, x_{n+k})\ , u\ v\epsilon\ (0, k) and\ r \neq v$$

where $\alpha_j(x)y_{n+j}$ represent equations and $\beta_j(x)f_{n+j}$ represent collocation equations

1.2 Definition 2

A method (2) is said to be Zero stable if no root of the first characteristics polynomial

$$\rho(r) = \sum_{j=0}^{k} \alpha_j\, r^j$$

has modulus greater than one and if every root with one is simple.

1.3 Definition 3

Every Linear Multistep method of Order $P > 1$ is convergent.

2 The Multi-step Collocation Method

Consider the collocation method defined for the step $[\, x_n,\ x_{n+k}]$ by

$$U(x) = y(x) = \sum_{j=0}^{t-1} Q_j(x) y_{n+j} + h^2 \sum_{j=0}^{m-1} \varphi_j(x) f\left(\bar{x}_j y(\bar{x}_j)\right) \quad (3)$$

where t and m denotes respectively the number of interpolation and collocation points used. We assume that $\emptyset_j(x), j = 0,..t-1$ and $\psi_j(x), j = 0,..m-1$ can be represented in polynomial form

$$U(x) = y(x) = \sum_{j=0}^{t+m-1} \emptyset_{j,i+1} x^i, \qquad h^2 \sum_{i=0}^{t+m-1} \psi_{j,j+1} x^i \tag{4}$$

Then $U(x)$ satisfy the conditions

$$U(x_{n+1}) = y_{n+1}, i = 0,..t-1 \tag{5}$$

$$U''(\bar{x}_j) = f\left(\bar{x}_j y(\bar{x}_j)\right), \quad j = 0,..m-1 \tag{6}$$

With the following conditions on

$\emptyset_j(x), j = 0,..t-1$ and $\psi_j(x), j = 0,..m-1$

$$\emptyset_j(x_{n+j}) = \partial_{ij}, j = 0,..m-1, i = 0,...m-1 \tag{7}$$

$$h^2 \psi''_j(\bar{x}_j) = \partial_{ij}, j = 0,..m-1, i = 0,...m-1 \tag{8}$$

2.1 Theorem 1

Let I denote the identity matrix of dimension $(m+t)$ x $(m+t)$ and consider the matrices C and D. Then

$i.$ $DC = I$

ii $\bar{y}(x) = \sum_{i=0}^{t+m-1} \left(\sum_{j=0}^{t-1} C_{i+1,j+1} y_{n+j} + \sum_{j=0}^{m-1} C_{i+1,j+1} f_{n+j} \right) x^i$

(See [10])

Next, writing the equation (5) through equation (8) in matrix equation form, we have

$$DC = I \tag{9}$$

where I is the identity matrix of dimension $(m+t)$ x $(m+t)$ and

$$D = \begin{bmatrix} 1 & . & . & x_n & x_n^2 & . & . & & & & x_n^{t+m-1} \\ 1 & . & . & x_{n+1} & x_{n+1}^2 & & . & & & & x_{n+1}^{t+m-1} \\ . & & . & . & . & & . & . & . & . & . \\ 1 & & & x_{n+1} & x_{n+1}^2 & . & . & x_{n+1}^{t+m-1} & & & \\ 0 & & . & 0 & 2 & . & & (t+m-1)(t+m-2)\bar{x}_0^{t+m-3} & & & \\ . & . & & . & . & & & . & . & . & . \\ . & . & & . & . & & & . & . & . & . \\ 0 & & . & 0 & 2 & . & & (t+m-1)(t+m-2)\bar{x}_{n-2}^{t+m-3} & & & \end{bmatrix} \tag{10}$$

$$C = \begin{bmatrix} \emptyset_{1,0} & \emptyset_{1,1} & . & . & \emptyset_{t-1,1} & h^2\psi_{0,1} & . & . & . & h^2\psi_{m-1,1} \\ \emptyset_{2,1} & \emptyset_{1,2} & . & . & \emptyset_{t-1,2} & h^2\psi_{0,2} & . & . & . & h^2\psi_{m-1,2} \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & \\ . & . & . & . & . & . & . & . & . & \\ \emptyset_{0,t+m} & \emptyset_{1,t+m} & . & . & \emptyset_{t-1,t+m} & h^2\psi_{0,t+m} & . & . & . & h^2\psi_{m-1,t+m} \end{bmatrix} \tag{11}$$

The columns of C which gives the continuous coefficients $\phi_j(x), j = 0,..t-1$ and $\psi_j(x), j = 0,...m-1$ can be obtained from the corresponding columns of D^{-1}. Thus we have explicitly (3) in the form

$$U(x) = (y_n \dots . y_{n+t-1}, f_n, \dots f_{n+m-1})C^T(1, x, \dots x^{t+m-1})^T \tag{12}$$

T denotes the 'transpose of'

3 Methodology

We construct an approximate solution to (1) in the form

$$\boldsymbol{y}(x) = \sum_{j=0}^{t+m-1} \alpha_j x^j \tag{13}$$

$$\boldsymbol{y}'(x) = \sum_{j=1}^{t+m-1} j\alpha_j x^{j-1} \tag{14}$$

$$\boldsymbol{y}''^{(x)} = \sum_{j=2}^{t+m-1} j(j-1)\alpha_j x^{j-2} = f(x, y, y') \tag{15}$$

Equation (13) and (15) is our Interpolation and Collocation equation for this method, where α_j are the parameters to be determined and $k = 5$. We Interpolate (13) at

$x = x_{n+j}, j = 0,1$ and collocate (15) at $x = x_{n+j}, j = 0,1,2,3,4,\frac{13}{3},\frac{9}{2}$ $and \frac{14}{3}$ yields the following system of non linear equations of the form

$$\sum_{j=0}^{t+m-1} \alpha_j \, x^j = y_{n+i} \qquad i = 0,1 \tag{16}$$

$$\sum_{j=0}^{t+m-1} j(j-1)\alpha_j \, x^{j-2} = f_{n+i} \qquad i = 0,1,2,3,4,\frac{13}{3},\frac{9}{2} \; and \; \frac{14}{3} \tag{17}$$

Our proposed Continuous formula is of the form

$$y(x) = \alpha_0(x)y_n + \alpha_1(x)y_{n+1} + h^2\Big(\beta_0(x)f_n + \beta_1(x)f_{n+1} + \beta_2(x)f_{n+2} + \beta_3(x)f_{n+3} + \beta_4(x)f_{n+4} + \beta_{\frac{13}{3}}(x)f_{n+\frac{13}{3}} + \beta_{\frac{9}{2}}(x)f_{n+\frac{9}{2}} + \beta_{\frac{14}{3}}(x)f_{n+\frac{14}{3}}\Big) \tag{18}$$

When using Maple 13 Mathematical Software to analysis equation (16) and (17) to obtain the values of $\alpha_j, j = 0,1,(k+3)$. Hence with some Mathematical manipulations, we obtained the Continuous formulation of the form

$$\begin{aligned} y(x) = {} & \left[\frac{h-\xi}{h}\right] y_n + \left[\frac{\xi}{h}\right] y_{n+1} + \left[-\frac{7334605}{33022080}h(\xi) + \frac{39312}{78624}(\xi)^2 - \frac{108132}{235872}\frac{(\xi)^3}{h} + \frac{117812}{471744}\frac{(\xi)^4}{h^2} - \frac{67217}{786240}\frac{(\xi)^5}{h^3} + \frac{21973}{1179360}\frac{(\xi)^6}{h^4} - \frac{4153}{1651104}\frac{(\xi)^7}{h^5} + \frac{47}{244608}\frac{(\xi)^8}{h^6} - \frac{1}{157248}\frac{(\xi)^9}{h^7}\right] f_n \\ & + \left[-\frac{2302739}{3880800}h(\xi) + \frac{19656}{13860}\frac{(\xi)^3}{h} - \frac{68820}{55440}\frac{(\xi)^4}{h^2} + \frac{24496}{46200}\frac{(\xi)^5}{h^3} - \frac{3645}{27720}\frac{(\xi)^6}{h^4} + \frac{1874}{97020}\frac{(\xi)^7}{h^5} - \frac{135}{86240}\frac{(\xi)^8}{h^6} + \frac{1}{18480}\frac{(\xi)^9}{h^7}\right] f_{n+1} \\ & + \left[\frac{836199}{940800}h(\xi) - \frac{19656}{6720}\frac{(\xi)^3}{h} + \frac{44238}{13440}\frac{(\xi)^4}{h^2} - \frac{36787}{22400}\frac{(\xi)^5}{h^3} + \frac{3043}{6720}\frac{(\xi)^6}{h^4} - \frac{3379}{47040}\frac{(\xi)^7}{h^5} + \frac{387}{62720}\frac{(\xi)^8}{h^6} - \frac{1}{4480}\frac{(\xi)^9}{h^7}\right] f_{n+2} \\ & + \left[-\frac{519359}{302400}h(\xi) + \frac{6552}{1080}\frac{(\xi)^3}{h} - \frac{31676}{4320}\frac{(\xi)^4}{h^2} + \frac{14356}{3600}\frac{(\xi)^5}{h^3} - \frac{2567}{2160}\frac{(\xi)^6}{h^4} + \frac{1523}{7560}\frac{(\xi)^7}{h^5} - \frac{41}{2240}\frac{(\xi)^8}{h^6} + \frac{1}{1440}\frac{(\xi)^9}{h^7}\right] f_{n+3} \\ & + \left[\frac{377147}{40320}h(\xi) - \frac{9828}{288}\frac{(\xi)^3}{h} + \frac{24576}{576}\frac{(\xi)^4}{h^2} - \frac{23309}{960}\frac{(\xi)^5}{h^3} + \frac{10977}{1440}\frac{(\xi)^6}{h^4} - \frac{2749}{2016}\frac{(\xi)^7}{h^5} + \frac{117}{896}\frac{(\xi)^8}{h^6} - \frac{1}{192}\frac{(\xi)^9}{h^7}\right] f_{n+4} \\ & + \left[-\frac{27996759}{1019200}h(\xi) + \frac{367416}{3640}\frac{(\xi)^3}{h} - \frac{1851660}{14560}\frac{(\xi)^4}{h^2} + \frac{2662308}{36400}\frac{(\xi)^5}{h^3} - \frac{169371}{7280}\frac{(\xi)^6}{h^4} + \frac{107649}{25480}\frac{(\xi)^7}{h^5} - \frac{83835}{203840}\frac{(\xi)^8}{h^6} + \frac{243}{14560}\frac{(\xi)^9}{h^7}\right] f_{n+\frac{13}{3}} \\ & + \left[\frac{2654272}{99225}h(\xi) - \frac{279552}{2835}\frac{(\xi)^3}{h} + \frac{353408}{2835}\frac{(\xi)^4}{h^2} - \frac{340352}{4725}\frac{(\xi)^5}{h^3} + \frac{65344}{2835}\frac{(\xi)^6}{h^4} - \frac{83648}{19845}\frac{(\xi)^7}{h^5} + \frac{304}{735}\frac{(\xi)^8}{h^6} - \frac{16}{945}\frac{(\xi)^9}{h^7}\right] f_{n+\frac{9}{2}} \\ & + \left[-\frac{25838919}{3449600}h(\xi) + \frac{682344}{24640}\frac{(\xi)^3}{h} - \frac{1730646}{49280}\frac{(\xi)^4}{h^2} + \frac{5022081}{246400}\frac{(\xi)^5}{h^3} - \frac{161595}{24640}\frac{(\xi)^6}{h^4} + \frac{208251}{172480}\frac{(\xi)^7}{h^5} - \frac{82377}{689920}\frac{(\xi)^8}{h^6} + \frac{243}{49280}\frac{(\xi)^9}{h^7}\right] f_{n+\frac{14}{3}} \end{aligned} \tag{19}$$

where $\xi = (x - x_n)$. .

Evaluating (19) at $x = x_{n+j}$ $j = 2,3,4,\frac{13}{3},\frac{9}{2},\frac{14}{3}$ .Also the first derivative of (19) at $x = x_n$ yield the following discrete schemes.

$$y_{n+2}-2y_{n+1}+y_n=\frac{h^2}{7.119560448\times10^{11}}\Big[46087475280f_n+674899767400f_{n+1}-184565220800f_{n+2}+559323404600f_{n+3}-3206238956000f_{n+4}+9490509595000f_{n+\frac{13}{3}}-9271465083000f_{n+\frac{9}{2}}+2603405062000f_{n+\frac{14}{3}}\Big]$$

$$y_{n+3}-3y_{n+1}+2y_n=\frac{h^2}{7.119560448\times10^{11}}\Big[90351362640f_n+1412293821000f_{n+1}+238037599800f_{n+2}+1104450067000f_{n+3}-587353142400f_{n+4}+17349758280000f_{n+\frac{13}{3}}-16941432670000f_{n+\frac{9}{2}}+4755941092000f_{n+\frac{14}{3}}\Big]$$

$$y_{n+4}-4y_{n+1}+3y_n=\frac{h^2}{7.119560448\times10^{11}}\Big[134919634100f_n+2145147339000f_{n+1}+734159265800f_{n+2}+2230654346000f_{n+3}-8602590316000f_{n+4}+25716548090000f_{n+\frac{13}{3}}-25155564990000f_{n+\frac{9}{2}}+7068462907000f_{n+\frac{14}{3}}\Big]$$

$$y_{n+\frac{13}{3}}-\frac{13}{3}y_{n+1}+\frac{10}{3}y_n=\frac{h^2}{1.724729948\times10^{16}}\Big[3.627708309\times10^{15}f_n+5.789221212\times10^{16}f_{n+1}+2.173824872\times10^{16}f_{n+2}+6.372466434\times10^{16}f_{n+3}-2.245635059\times10^{17}f_{n+4}+6.83327799\times10^{17}f_{n+\frac{13}{3}}-6.695583169\times10^{17}f_{n+\frac{9}{2}}+1.883750199\times10^{17}f_{n+\frac{14}{3}}\Big]$$

$$y_{n+\frac{9}{2}}-\frac{9}{2}y_{n+1}+\frac{7}{2}y_n=\frac{h^2}{5.290107863\times10^{13}}\Big[1.167789974\times10^{13}f_n+1.866548017\times10^{14}f_{n+1}+7.273907537\times10^{13}f_{n+2}+2.103071877\times10^{14}f_{n+3}-7.132527412\times10^{14}f_{n+4}+2.190536779\times10^{15}f_{n+\frac{13}{3}}-2.146214604\times10^{15}f_{n+\frac{9}{2}}+6.04147596\times10^{14}f_{n+\frac{14}{3}}\Big]$$

$$y_{n+\frac{14}{3}}-\frac{14}{3}y_{n+1}+\frac{11}{3}y_n=\frac{h^2}{1.123154382\times10^{15}}\Big[2.596335973\times10^{14}f_n+4.155842433\times10^{15}f_{n+1}+1.673072259\times10^{15}f_{n+2}+4.780361009\times10^{15}f_{n+3}-1.5662665732\times10^{15}f_{n+4}+4.851910268\times10^{16}f_{n+\frac{13}{3}}-4.750512138\times10^{16}f_{n+\frac{9}{2}}+1.338897643\times10^{16}f_{n+\frac{14}{3}}\Big]$$

$$hy'_n-y_{n+1}+y_n=\frac{h^2}{3632428800}\Big[-806806550f_n-2155363704f_{n+1}+3228564339f_{n+2}-6238540308f_{n+3}+33977173230f_{n+4}-99780449076f_{n+\frac{13}{3}}+97167589376f_{n+\frac{9}{2}}-27208381707f_{n+\frac{14}{3}}\Big] \quad (20)$$

Equation (20) is our Seven Point Block schemes which is of Order $[8,8,8,8,8,8,8]^T$ with Error Constants of

$$\left[-\frac{54563}{38102400},-\frac{623123}{33868800},-\frac{1855033}{6350400},-\frac{258731308537}{399983754240},-\frac{222245827}{235929600},-\frac{48345687137}{35712835200},8237801\right]^T$$

Also the first derivative of (19) is evaluated at $x=x_{n+j}\ j=1,2,3,4,\frac{13}{3},\frac{9}{2},\frac{14}{3}$ and yields the following discrete schemes.

$$y'_{n+1} = -\frac{1}{3632428800}\frac{1}{h}\Big[3632428800y_n - 3632428800y_{n+1} - 253301290h^2f_n - 2474064216h^2f_{n+1} + 2433746601h^2f_{n+2} - 4472439972h^2f_{n+3} + 23848354530h^2f_{n+4} - 69714587844h^2f_{n+\frac{13}{3}} + 67752914944h^2f_{n+\frac{9}{2}} - 18936837153h^2f_{n+\frac{14}{3}}\Big]$$

$$y'_{n+2} = -\frac{1}{3632428800}\frac{1}{h}\Big[3632428800y_n - 3632428800y_{n+1} - 222965930h^2f_n - 3805084296h^2f_{n+1} - 1094203539h^2f_{n+2} - 1449139692h^2f_{n+3} + 10185665490h^2f_{n+4} - 31028045004h^2f_{n+\frac{13}{3}} + 30661689344h^2f_{n+\frac{9}{2}} - 8696559573h^2f_{n+\frac{14}{3}}\Big]$$

$$y'_{n+3} = -\frac{1}{3632428800}\frac{1}{h}\Big[3632428800y_n - 3632428800y_{n+1} - 227988970h^2f_n - 3731748696h^2f_{n+1} - 2584317879h^2f_{n+2} - 4720367652h^2f_{n+3} + 16753586850h^2f_{n+4} - 48236498244h^2f_{n+\frac{13}{3}} + 46666706944h^2f_{n+\frac{9}{2}} - 13000444353h^2f_{n+\frac{14}{3}}\Big]$$

$$y'_{n+4} = -\frac{1}{3632428800}\frac{1}{h}\Big[3632428800y_n - 3632428800y_{n+1} - 226975210h^2f_n - 3743982216h^2f_{n+1} - 2497136499h^2f_{n+2} - 6123249132h^2f_{n+3} + 10557557010h^2f_{n+4} - 37679324364h^2f_{n+\frac{13}{3}} + 37784141824h^2f_{n+\frac{9}{2}} - 10784532213h^2f_{n+\frac{14}{3}}\Big]$$

$$y'_{n+\frac{13}{3}} = -\frac{1}{2648040595200}\frac{1}{h}\Big[2648040595200y_n - 2648040595200y_{n+1} - 1654748404410h^2f_n - 2729251885464h^2f_{n+1} - 1821084707871h^2f_{n+2} - 4459927059588h^2f_{n+3} + 7344019052370h^2f_{n+4} - 28266476250276h^2f_{n+\frac{13}{3}} + 27872228274176h^2f_{n+\frac{9}{2}} - 7924854864537h^2f_{n+\frac{14}{3}}\Big]$$

$$y'_{n+\frac{9}{2}} = -\frac{1}{464950886400}\frac{1}{h}\Big[464950886400y_n - 464950886400y_{n+1} - 29054370290h^2f_n - 479212374888h^2f_{n+1} - 319738857867h^2f_{n+2} - 783154448076h^2f_{n+3} + 1291208888970h^2f_{n+4} - 5003815488012h^2f_{n+\frac{13}{3}} + 4852672428032h^2f_{n+\frac{9}{2}} - 1388709323469h^2f_{n+\frac{14}{3}}\Big]$$

$$y'_{n+\frac{14}{3}} = -\frac{1}{2648040595200}\frac{1}{h}\Big[2648040595200y_n - 2648040595200y_{n+1} - 165475759130h^2f_n - 2729242029384h^2f_{n+1} - 1821139611291h^2f_{n+2} - 4459666519308h^2f_{n+3} + 7340847163650h^2f_{n+4} - 28401623664876h^2f_{n+\frac{13}{3}} + 27270612402176h^2f_{n+\frac{9}{2}} - 8067814461837h^2f_{n+\frac{14}{3}}\Big] \quad (21)$$

Equation (21) is of Order $[8,8,8,8,8,8,8]^T$ with Error constants

$$\left[-\frac{131824693}{24}, -\frac{24915605}{3}, -\frac{1606941479}{8}, -\frac{7883145413}{3}, -\frac{94420457413997}{24}, -\frac{15519804655355}{16}, -7662557801327\right]^T$$

4 Block Analysis of the New Method

We arrange the right hand side of equation (20) in Matrix form as

$$A^0 = \begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ -3 & 0 & 1 & 0 & 0 & 0 & 0 \\ -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{13}{3} & 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{9}{2} & 0 & 0 & 0 & 0 & 1 & 0 \\ -\frac{14}{3} & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ y_{n+4} \\ y_{n+\frac{13}{3}} \\ y_{n+\frac{9}{2}} \\ y_{n+\frac{14}{3}} \end{bmatrix}, A^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{10}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{7}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{11}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} y_{n-\frac{11}{3}} \\ y_{n-\frac{8}{3}} \\ y_{n-\frac{5}{3}} \\ y_{n-\frac{2}{3}} \\ y_{n-\frac{1}{3}} \\ y_{n-\frac{1}{6}} \\ y_n \end{bmatrix}$$

Multiply$A^0$and A^1 by the inverse Matrix of A^0 for Normalization of the block method to obtain

$$B^0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ y_{n+4} \\ y_{n+\frac{13}{3}} \\ y_{n+\frac{9}{2}} \\ y_{n+\frac{14}{3}} \end{bmatrix}, B^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & \frac{14}{3} \\ 0 & 0 & 0 & 0 & 0 & -2 & \frac{28}{3} \\ 0 & 0 & 0 & 0 & 0 & -3 & 14 \\ 0 & 0 & 0 & 0 & 0 & -\frac{10}{3} & \frac{140}{9} \\ 0 & 0 & 0 & 0 & 0 & -\frac{7}{2} & \frac{49}{3} \\ 0 & 0 & 0 & 0 & 0 & -\frac{11}{3} & \frac{154}{9} \\ 0 & 0 & 0 & 0 & 0 & -1 & \frac{14}{3} \end{bmatrix} \begin{bmatrix} y_{n-\frac{11}{3}} \\ y_{n-\frac{8}{3}} \\ y_{n-\frac{5}{3}} \\ y_{n-\frac{2}{3}} \\ y_{n-\frac{1}{3}} \\ y_{n-\frac{1}{6}} \\ y_n \end{bmatrix}$$

$\rho(R) = det[RB^0 - B^1] = 0$

Then $R^7 + \frac{11}{3}R^5 = 0$, It implies that $R_1 = R_2 = R_3 = R_4 = R_5 = 0$, $R_6 = \frac{1}{3}\, i\sqrt{3}$ and $R_{7=} -\frac{1}{3}\, i\sqrt{3}$. Following the Definition 2, the Block method propose is Zero stable and Convergent since the Order of the schemes is > 1 .

5 Implementation Strategies

In the process of obtaining the solution of General Second Order ODEs directly, equation (21) is substituted in equation (20) at $n = 0$ simultaneously solve to give Seven point solutions of $y_1, y_2, y_3, y_4, y_{\frac{13}{3}}, y_{\frac{9}{2}}$ and $y_{\frac{14}{3}}$ at once. The advancement of the integration is done with the main Scheme together with the Hybrid Schemes in equation (20) without a derivative Scheme present in it at $n = 4,8.12, ..$

6 Numerical Experiments

Three Numerical Experiments of two linear and one non linear problem were used to ascertain the accuracy of the method.

Example 1

$y'' + \frac{6}{x}\, y' + \frac{4}{x^2}\, y = 0$

$y(1) = 1, y'(1) = 1, \qquad h = \dfrac{0.1}{32} \quad x > 0$

Theoretical solution is $y(x) = \frac{5}{3x} - \frac{2}{3x^4}$

Example 2

$y'' - 3y' = 8e^{2x}$

$y(0) = 1\, , y'(0) = 1\ \ , h = 0.005$

Theoretical solution is $y(x) = -4e^{2x} + 3e^{3x} + 2$

Example 3

$y'' - 3y' = 8e^{2xy}$

$y(0) = 1\, , y'(0) = 1\, , h = 0.005$

No theoretical solution

Table 1. Table of result for example 1

x	Theoretical solution	Block method [9]	New block method
1.003125	1.003076526	1.003114880	1.0030766905
1.00625	1.006057503	1.006132507	1.00605684265
1.009375	1.008944993	1.009050907	1.0089405789
1.0125	1.011741018	1.011876494	1.01172802434
1.015625	1.014447543	1.014603110	1.014431165439
1.01875	1.017066494	1.017252866	1.017038197167
1.021875	1.019599755	1.019795810	1.01954923805
1.025	1.022049164	1.022270209	1.02201055468
1.028125	1.024416519	1.024622147	1.02434160973
1.03125	1.026703578	1.026981486	1.026557694498

Table 2. Absolute error of problem 1

Block method [9]	New block method
3.8354 E (-05)	1.645 E (-07)
7.5004 E (-05)	6.6035 E (-07)
1.05926 E (-04)	4.4141 E (-06)
1.35476 E (-04)	1.299366 E (-05)
1.55567 E (-04)	1.6377561 E (-05)
1.863726 E (-04)	2.8296833 E (-05)
1.96055 E (-04)	5.051695 E (-05)
2.21045 E (-04)	3.860932 E (-05)
2.0562 E (-04)	7.490927 E (-05)
2.77908 E (-04)	1.458835 E (-04)

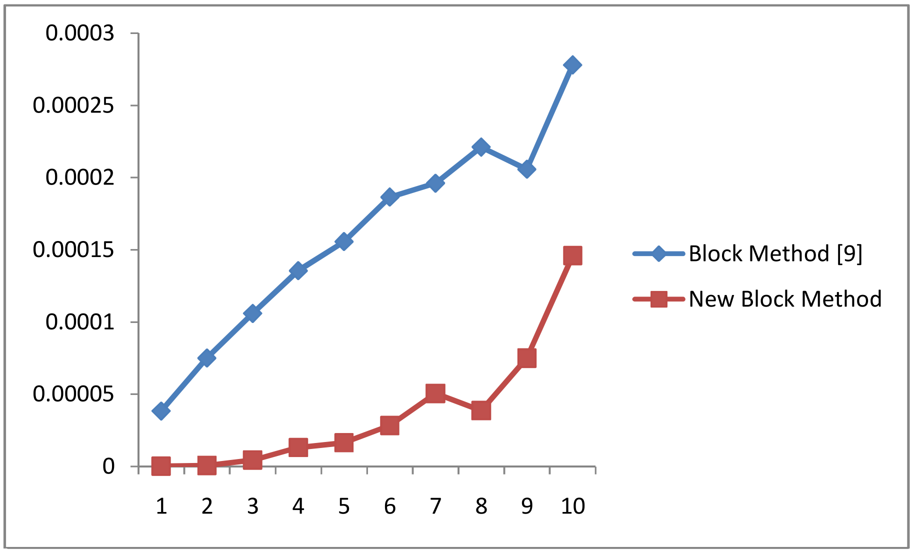

Fig. 1. Error graph of Table 2

Table 3. Table of result for example 2

x	Theoretical solution	Block method [7]	New block method
0.005	1.005138526	1.005139114	1.0051388419
0.01	1.010558242	1.010557205	1.0105569711
0.015	1.016265444	1.016255068	1.0162567886
0.02	1.022266643	1.022226977	1.0222407282
0.025	1.028568067	1.028508035	1.02853411642
0.03	1.035176665	1.035010659	1.03511676083
0.04	1.049342284	1.048928801	1.04925342567

Table 4. Absolute error of problem 2

Block method [7]	New block method
5.8849 E(-07)	3.159 E (-07)
1.03675 E-06	1.2709 E (-06)
1.03759 E-05	8.6554 E (-06)
3.95659 E-05	2.59148 E (-05)
5.97171 E-05	3.395058 E (-05)
1.66006 E-04	5.990417 E (-05)
4.13483 E-04	8.885833 E (-05)

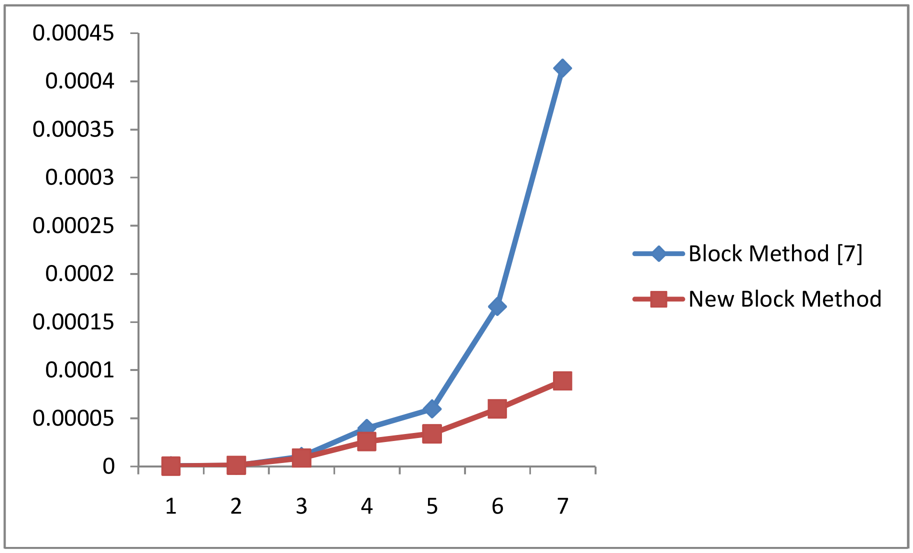

Fig. 2. Error graph of Table 4

Table 5. Table of result for example 3

x	**Block method [7]**	**New block method**
0.005	1.005139120	1.0051388451
0.01	1.010557226	1.0105569851
0.015	1.016255105	1.01625686111
0.02	1.02222703	1.0222409615
0.025	1.028508106	1.0285346996
0.03	1.035010745	1.035118000
0.04	1.048928909	1.049257509

7. Discussion of Results

The three problems tested with this algorithm of which the first two are linear and the third one is non linear general second order ordinary differential equations (ODEs).

Table 1 displaces the numerical results of problem1 and Table 2 shows their approximate errors of the method while figure 1 is the error graph of problem1.

Tables 3 and 4 displace the approximate solution of example 2 together with their absolute errors while figure 2 is the error graph of problem 2. Table 5 shows only the approximate solution of problem 3 at two different Block methods since its theoretical solution does not exist.

8 Conclusion

This paper provides an implicit linear multistep Hybrid Block method of Uniform order 8 at k =5, for direct approximate solution of general Second Order Differential Equation (ODEs) of the form $y'' = f(x, y, y')$. Interestingly, all the Discrete schemes in the Block method were gotten from the single Continuous Formulation (CF) together with its first derivative, which were of the Uniform Order. Numerical experiments of linear and non linear problems were used to demonstrate the efficiency of the new method. The method is self starting and results obtained were in block form which speed up the computational processes.

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Competing interests

Author has declared that no competing interests exist.

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