

Article

Formulating the Concept of an Investment Strategy Adaptable to Changes in the Market Situation

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Abstract: The study aims to develop a dynamic model for the management of a strategic investment portfolio, taking into account the impact of crisis processes on asset value. A mathematical model of a dynamic portfolio strategy is developed, and guidelines for framing a long-term investment strategy based on the current state of the investment market are formalized. An efficient method of long-term ensemble forecasting to increase the accuracy of predicting financial time series is elaborated. A methodology for constructing and rebalancing a dynamic strategic investment portfolio based on a changing portfolio strategy that results from assessing the current market state and forecast is developed. The obtained strategic portfolio model has been estimated empirically based on historical data and its rate-of-return characteristics have been compared with those of the existing conventional models used in strategic investment.

Keywords: investment strategy; innovation; portfolio model



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1. Introduction

Choosing the right investment mechanism is one of the main tasks for any investor, requiring careful analysis and research of all available information (Yerznkyan et al. 2021). Therefore, no investor knows exactly whether their expectations regarding the return on particular equity will be met, but they need to build their strategy in such a way as to eliminate the damage as much as possible in the event of an unfavorable outcome. The development of a universal investment vehicle could facilitate the activities of investors; however, it seems a challenging task thus far to build a unified model that covers the whole variety of factors. Creating an investment portfolio is an urgent issue in the current conditions of the rapid evolution of the stock market and is growing interest in investment from all participants in the economy.

The securities portfolio acts as an integral object of management. To solve the problem of the optimal securities portfolio means determining the optimal allocation of the asset shares in the portfolio, such that it meets the investor's goals. In 1952, in *The Journal of Finance*, Harry Max Markowitz, Professor of the University of Chicago, published his work, "Portfolio Selection" (Markowitz 1952), from which the theory of portfolio investment originates. In the article, H. Markowitz suggests a mathematical model for forming an optimal portfolio of securities.

A foundational step in the development of portfolio analysis was made by R. Roll. He developed a regression method for investment portfolio optimization known as Roll's rebalancing (Roll 1977). The ratio between the geometric and arithmetic mean rates of any set of the return components is used as a portfolio return indicator. In this case, all the components being equal, a rebalanced portfolio always outperforms an unbalanced one.

A substantial contribution to the study of portfolio strategy, dynamics and optimization of the investment portfolio was made by Dai Pra, P., and Tolotti, M. (Dai Pra and Tolotti 2009). The authors provided mathematical proof that a set of asset groups is more stable than each asset individually.

The authors Li, B., Sun, Y., Aw, G., and Teo, K. L. (Li et al. 2019) suggest using the Pareto equilibrium point to find the optimal static portfolio in a steadily growing market. The mathematical model is fully justified but has not been tested in practice. The authors argue that under certain conditions, such a portfolio may perform better than the minimum-risk portfolio by H. Markowitz.

The authors de Leeuw, T., Gilsing, V., and Duysters, G. (de Leeuw et al. 2019) compare a static, state-regulated portfolio model and five diverse, dynamic portfolio models with different dynamicity levels and find that despite a more than twofold difference in return demonstrated by dynamic models, a dynamic portfolio is preferable to a static portfolio which is recommended or regulated by the state.

The authors Bi, J., and Cai, J. (Bi and Jun 2019) recommend using an equilibrium portfolio strategy with constraints, and proposed to introduce the upper and lower VaR control limits that determine the need to change the equilibrium point of the portfolio strategy.

The authors Hadhri, S., and Ftiti, Z. (Hadhri and Ftiti 2019) suggest that for portfolio optimization, the measure of skewness should be used as a criterion, along with the classical risk/return pair. The authors note that for a static portfolio, skewness is an important element that characterizes asset instability.

The authors Jeon, J., and Shin, Y. H. (Jeon and Shin 2019) address the choice of strategy in marginal markets under a finite investment horizon and offer a mathematically sound model of gradient optimization, which allows achieving minimal risk with minimal loss of a static portfolio.

The authors Wei, J., Bu, B., and Liang, L. (Wei et al. 2012) propose the simplest two-mode dynamic portfolio model that has a high-risk and low-risk mode and switches the strategy depending on arbitrarily determined external risk factors.

The authors Benita, F., López-Ramos, F., and Nasini, S. (Benita et al. 2019) present a four-level dynamic portfolio model that switches the strategy used depending on arbitrarily determined external risk factors.

The study (Lu et al. 2020) aims to use both the Sharpe ratio (SR) and the economic performance measure (EPM) to compare the performance of DCA and LS under both accumulative and disaccumulative approaches when the asset price is simulated to be in an uptrend.

The literature review has shown that the best investment strategy is one that adapts to external factors.

The authors Ma, G., Siu, C. C., and Zhu, S. P. (Ma et al. 2019) propose a dynamic portfolio that is optimized based on the analysis of the predicted market situation, instead of relying on the current trends. In fact, the authors recommend using the classic Merton portfolio, optimized based on the expected market conditions. The authors Lu, Y. N. et al. (Lu et al. 2018) suggest splitting the investment portfolio into several different portfolios with different strategies. In doing so, the method of strategic diversification is implemented. In fact, in this paper, the authors described a new method of portfolio diversification, namely, strategic diversification. Golosnoy, V., Gribisch, B., Seifert, M. I. (Golosnoy et al. 2019) suggest predicting the portfolio behavior by obtaining separate predictions for each asset share in the portfolio. Empirical evidence shows that the combined forecast of the asset shares is more accurate in practice than the forecast made for the entire portfolio.

In the above studies dedicated to the analysis of portfolio investment, almost no attention is paid to the specification of the strategy, nor to methods of scoring its performance, or to the adaptability of strategies. To date, the only traditionally used method of scoring a strategy's performance is simulation modeling.

Having formalized the components of an investment strategy, one can develop and apply quantitative methods of strategy evaluation.

In this regard, the scope of the in-depth scientific study was defined as follows: 1. observation of the methods of system analysis; 2. formalization of the construction

and evaluation of investment strategies; 3. synthesis of various approaches based on the refinement of classical and neoclassical theories, as well as the creation of models and methods of strategic investment taking into account the real state of the market with an allowance for the risk of crisis changes.

2. Literature Review

To date, many works have been concerned with the creation of investment portfolios using innovative mathematical tools of machine learning and fuzzy logic. Let us look at some of them.

In the work of Chen Shun and Ge Lei ([Chen and Ge 2021](#)), the strategy of an optimal investment portfolio selection based on a neural network model is investigated. A numerical comparison of the results obtained in the article with those of classic solutions demonstrates the effectiveness of the learning-based strategy. To select the optimal portfolio, in the work of Zhou Wei and Zeshui Xu ([Zhou and Xu 2018](#)), qualitative models for portfolio construction under uncertain conditions of a fuzzy environment were proposed. Guan Hao and Zhiyong An ([Guan and Zhiyong 2019](#)) propose a local adaptive learning system for selecting an online portfolio. In their article, Park, Hyungjun, Min Kyu Sim, and Dong Gu Choi ([Park et al. 2020](#)) suggest a novel portfolio-trading strategy in which an intelligent agent is trained to identify an optimal trading action by using deep Q-learning. In their study, Ma Yilin, Ruizhu Han, and Weizhong Wang ([Ma et al. 2021](#)) suggest optimizing the portfolio based on the prediction of asset returns. The authors use five models for predicting stock returns, which improves the quality of stock selection for constructing an investment portfolio. Francesco Cesarone, Carlo Mottura, Jacopo M Ricci, and Fabio Tardella ([Cesarone et al. 2020](#)) compared several popular investment portfolio selection models in terms of their sensitivity to noise. The results showed that the portfolios with the highest returns are the most sensitive to noise. Felipe Dias Paiva, Rodrigo Tomás Nogueira Cardoso, Gustavo Peixoto Hanaoka, and Wendel Moreira Duarte ([Paiva et al. 2019](#)) propose a decision-making model for day-trading investments on the stock market. The model is based on the support vector machine method and the mean-variance method for portfolio selection. In the work of Khedmati Majid and Pejman Azin ([Khedmati and Azin 2020](#)), an online portfolio-selection algorithm based on the pattern-matching principle is presented. In their work, Eom Cheoljun and Jong Won Park ([Eom and Park 2019](#)) developed a method for estimating a correlation matrix, on the basis of which a well-diversified portfolio was constructed. In their article, Jinhua Chang, Lin Sun, Bo Zhang, and Jin Peng ([Chang et al. 2020](#)) consider the uncertain multi-period portfolio selection problem in a situation where future security return rates are given by experts' estimations instead of historical data. The paper provides a case study that illustrates the effectiveness of the developed model. Xiaomin Gong, Changrui Yu, Liangyu Min, and Zhipeng Ge ([Gong et al. 2021](#)) propose a model for evaluating cross-efficiency scores of assets aimed at maximizing an investor's perceived utility (composed of utility, regret, and rejoice functions). To demonstrate the effectiveness of the proposed model, a real-world empirical application is presented. Xingying Yu, Yang Shen, Xiang Li, and Kun Fan ([Yu et al. 2020](#)) consider the mean-variance portfolio selection problem with uncertain model parameters. The authors formulate the mean-variance problem under the maxmin criterion. Using the Lagrangian method, an efficient portfolio is obtained. In their article, Puerto Justo, Moisés Rodríguez-Madrena, and Andrea Scozzari ([Puerto et al. 2020](#)) propose a linear programming approach to solve the portfolio-selection problem. When almost all underlying assets suddenly lose a certain part of their nominal value in a market crash, the diversification effect of portfolios under normal market conditions no longer works. Shushang Zhu, Wei Zhu, Xi Pei, and Xueting Cui ([Zhu et al. 2020](#)) assess the crash risk in portfolio management and investigate hedging techniques involving derivatives to optimize the investment portfolio. In the study by Mansour Nabil, Mohamed Sadok Cherif, and Walid Abdelfattah ([Mansour et al. 2019](#)), they developed an approach involving fuzzy parameters, where the possibility distributions are given by fuzzy numbers from the information supplied by the decision-making

environment (investor, analyst, financial market environment, etc.). The proposed financial portfolio selection model with fuzzy returns is used within the stock exchange market and discussed in comparison with other portfolio selection models. Guoli Mo, Chunzhi Tan, Weiguo Zhang, and Fang Liu (Mo et al. 2019) suggest forming an international portfolio of stock indices with spatiotemporal correlation. The research by Mishra Sasmita and Sudarsan Padhy (Mishra and Padhy 2019) proposes an efficient investment portfolio construction model using predictive analytics based on support vector regression. Gang Chu, Wei Zhang, Guofeng Sun, and Xiaotao Zhang (Chu et al. 2019) propose a new online portfolio selection algorithm based on the Kalman filter and anticorrelation. In the article by Borovička, Adam (Borovička 2020), a unique fuzzy multiple objective programming algorithm is proposed for creating a portfolio in conditions of uncertainty.

3. Methodology

To create an efficient modern portfolio model, we will develop a basic strategic concept. When creating the concept, we will rely on several generally accepted economic hypotheses.

Hypothesis H1: *Based on historical data and methods of multivariate regression analysis, it is possible to construct a strategic investment portfolio that will periodically change its investment strategy and break even regardless of the current market situation.*

Hypothesis H2: *The market state is stable or tends to be one in which supply and demand are balanced.*

Corollary to Hypothesis H2: we can break down the market state in the following terms:

1. Stagnation—a stable state;
2. Growth—a situation where demand exceeds supply;
3. Decline—a situation where supply exceeds demand.

Hypothesis H3: *The hypothesis of the identical character of crises. The phases of economic crises are always identical.*

3.1. The Concept of an Innovative Investment Strategy Adaptable to Changes in the Market Situation

To form the basic concept, we define a market as any non-empty set of traded assets. Such a system is characterized by the following conceptual approaches:

1. Combined market—part of the market information is distributed instantly, is publicly available, and directly reflected in the price of the asset; the other part is reflected with a delay or indirectly (Gataullin et al. 2020; Gorodetskaya et al. 2021; Yerznkyan et al. 2019).
2. Combined return—changes in the price of an asset can be considered as an aggregated stochastic process; the speculative preferences of investors (Sunchalin et al. 2019; Ivanyuk and Tsvirkun 2013).
3. Segmented market—any non-empty set of assets can simultaneously be considered as a market, a portfolio, or an asset itself if the set contains a single element.
4. Limited market—in market development, time constraints are determined by the forecast horizon.
5. Forecasted market—in market development, most trends are predictable.
6. Rational investor—the investor's interest is to achieve the maximum possible increase in the portfolio value during the forecast period with the minimum predicted risk.
7. Finite investment period—the duration of the investment period is determined by the forecast horizon.

According to the above concepts, the market states are different at different times. The differences are so great that there is no single investment strategy that would be profitable in any time period, under any market conditions. The main conceptual approach that

follows from the above is that the investment strategy must be changed in accordance with market developments.

The main advantages of this concept include:

- balance between disregarding instantaneous market changes and taking into account fundamental market factors;
- adaptability of the strategy;
- predictively justified approach to the optimal portfolio creation.

Let us form a general algorithm for implementing the strategy:

- Data acquisition;
- Market analysis;
- Portfolio strategy adjustment;
- Asset selection;
- Forecasting;
- Portfolio rebalancing.

Basic strategy: change the strategy at every rebalancing event. The strategy is formed by the market (Figure 1).

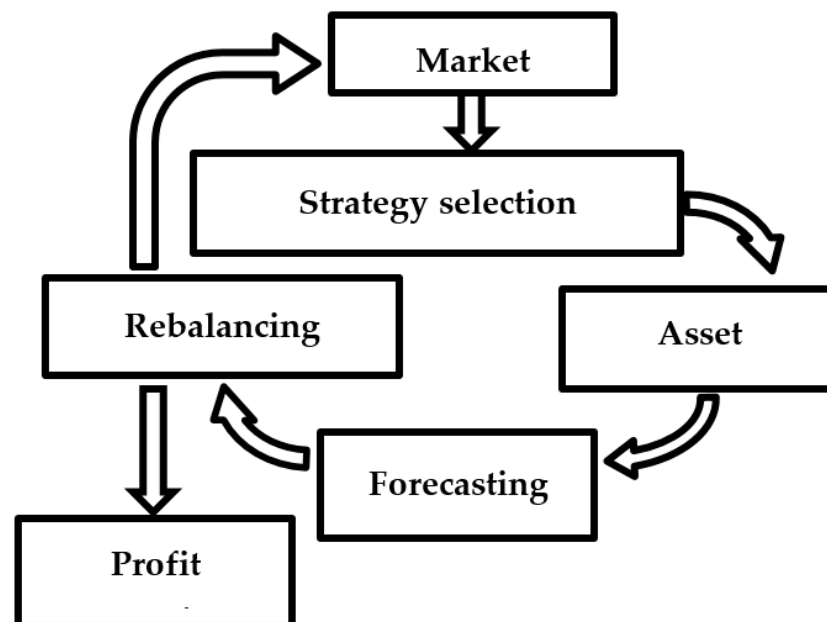


Figure 1. The concept of an innovative investment strategy adaptable to changes in the market situation.

The mathematical expression of this concept is the value of the optimal asset share of the portfolio, depending on the superposition of functions, as written below:

$$x_i = Optimization(Forecast(Strategy(Analythic(\gamma))), \quad (1)$$

where: $\{\gamma\}$ —a set of historical market data; x_i —the portfolio share of the asset.

3.2. Algorithm for Developing an Innovative Investment Strategy That Is Adaptable to Changes in the Market Situation and Has the Properties of an Open System

The algorithm for forming a dynamic strategy portfolio can be presented as Figure 2.

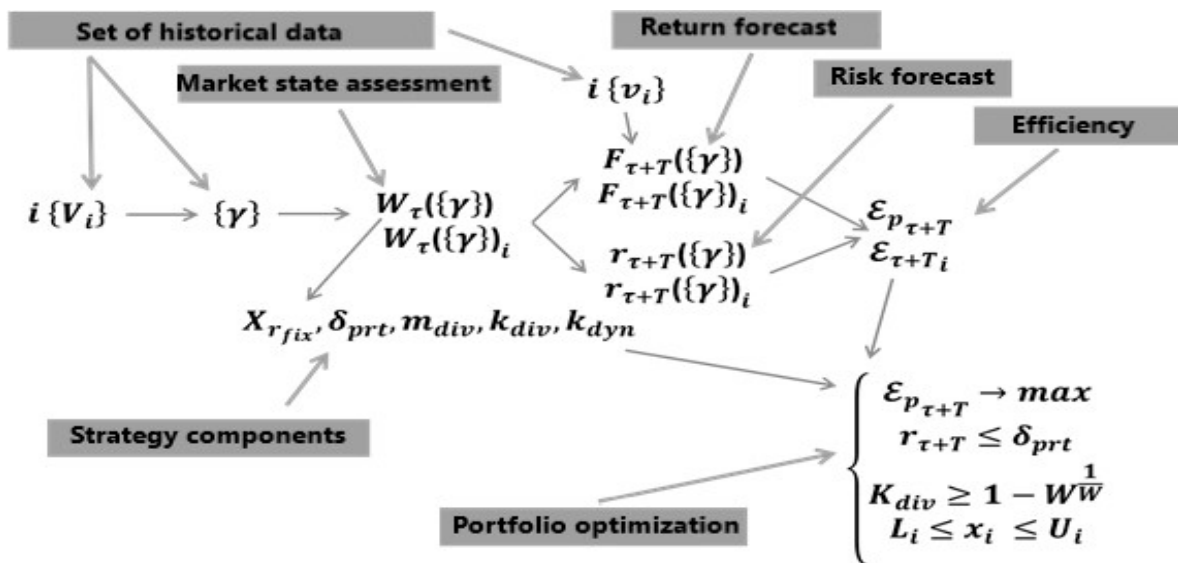


Figure 2. Portfolio construction algorithm for an adaptive strategy investment portfolio.

4. Methodology for the Development of an Adaptive Investment Strategy

Let us define a set of possible results of the state analysis function $W = Analythic(\{\gamma\})$ presented by the decrease of the potential profit: Speculative growth, Growth, Dynamic stagnation, Decline, Crisis decline (Figure 3).

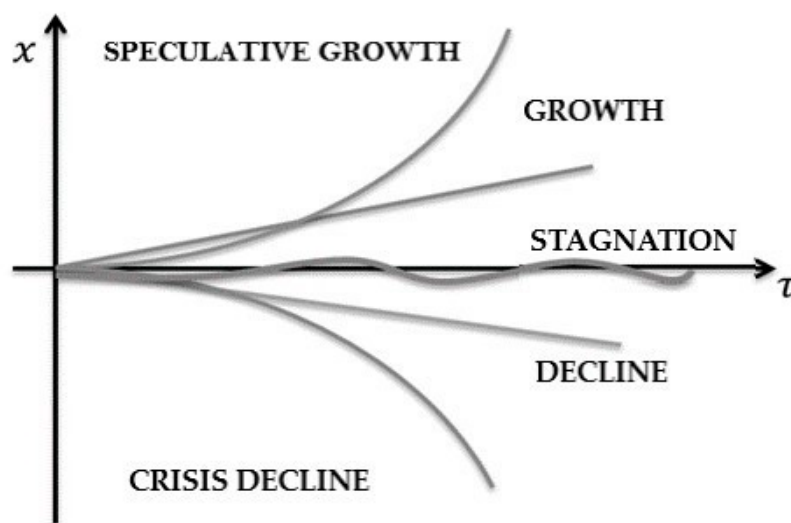


Figure 3. Possible results of the market analysis function.

To create an initial market assessment, we determine the boundaries of basic normalized states on the interval $[0; 1]$, taking into account that the state of dynamic stagnation is a constant (see Table 1).

To determine the market state at the time t for the beginning of the investment period T , we will use data on the market state for the preceding period T_{-1} .

Table 1. Boundaries of basic states.

Market State	Analythic($\{\gamma\}$)
Speculative growth	(0.75; 1]
Growth	(0.5; 0.75]
Dynamic stagnation	0.5
Decline	[0.25; 0.5)
Crisis decline	[0; 0.25)

4.1. Model of Dynamic Stagnation of the Asset Value

We define dynamic stagnation as a situation where the asset value $V_{\forall \tau \in T}$ lies in the price corridor of the $\sim 95\%$ -th probability: $M_T \pm 2\sigma_T$, provided that ΔM_T lies within the inflation-conditioned limits (Figure 4).

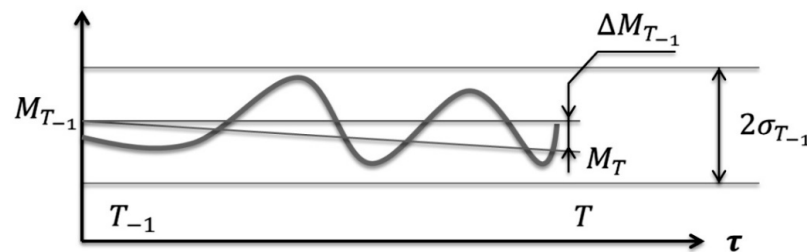


Figure 4. Dynamic stagnation model.

The width of the permitted movement corridor M_T is assumed to be equal to the sum of the inflationary factor k_I for the periods T_{-1} and $2\sigma_T$:

Then, the state of dynamic stagnation Equation (2) is expressed as 0.5:

$$\omega_c = 0.5 \cdot \theta(|k_I M_{T-1}| + 2\sigma_{T-1} - |\Delta M_{T-1}|), \tag{2}$$

where: ω_c —indicator of stagnation; θ —Heaviside function: $\theta(x) = \begin{cases} 0, & x < 0; \\ 1, & x \geq 0. \end{cases}$; k_I —inflationary factor; M_{T-1} —mathematical expectation $V_{\forall \tau \in T_{-1}}$ for T_{-1} ; σ_{T-1} —standard deviation $V_{\forall \tau \in T_{-1}}$ for T_{-1} .

4.2. The Model of Growth and Decline of the Asset Value

The direction of the return indicator for the investment period T can be defined as the difference in the values of the mathematical expectation ΔM of the portfolio market value for the current period T and the previous period T_{-1} : positive values indicate growth, negative values indicate a decrease (Figure 5)

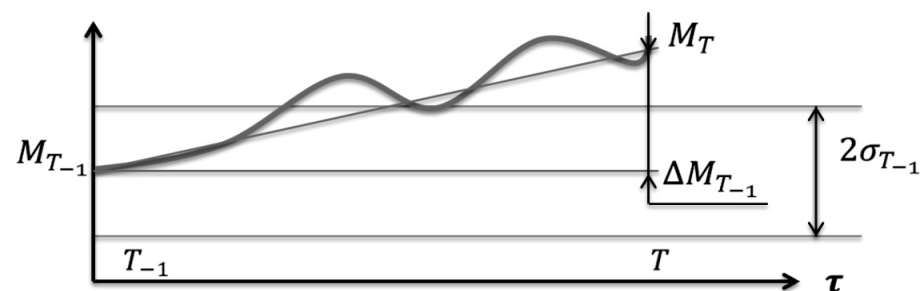


Figure 5. Growth and decline model.

Then, we will express the share of the market growth Equation (3) on the interval [0.25;0.75] as:

$$\omega_p = 0.5 + 0.25 \cdot \text{sgn}(\Delta M_{T-1}) \frac{|\Delta M_{T-1}|}{|\Delta M_T|_{max}}, \tag{3}$$

where:

ω_p —share of market growth (decline) [0.25;0.75];

sgn —Kronecker function: $\text{sgn}(x) = \begin{cases} -1, & x < 0; \\ 0, & x = 0; \\ 1, & x > 0. \end{cases}$

ΔM_{T-1} —change in the mathematical expectation for the period $T-1$;

$|\Delta M_T|_{max}$ —the largest change in the value for the period T achieved over the entire period of the asset's existence T_{uct} .

Thus, the state of a non-crisis market, in view of stagnation, can be expressed as Equation (4):

$$\omega_{pc} = 0.5 + \theta(|\Delta M_{T-1}| - |k_I M_{T-1}| - 2\sigma_{T-1}) \left(0.25 \cdot \text{sgn}(\Delta M_{T-1}) \frac{|\Delta M_{T-1}|}{|\Delta M_T|_{max}} \right) \tag{4}$$

where: ω_{pc} —non-crisis market state [0.25;0.75].

4.3. Asset Crisis Model

Let us define a crisis as an imbalance of positive and negative indicators that exceeds the statistically valid probability bounds set at 99.7%. Based on the three-sigma rule, all values of a normally distributed random variable lie within the interval $[M - 3\sigma; M + 3\sigma]$. Let us estimate the crisis state as a situation where the volatility indicator falls outside the statistically predicted range (Figure 6):

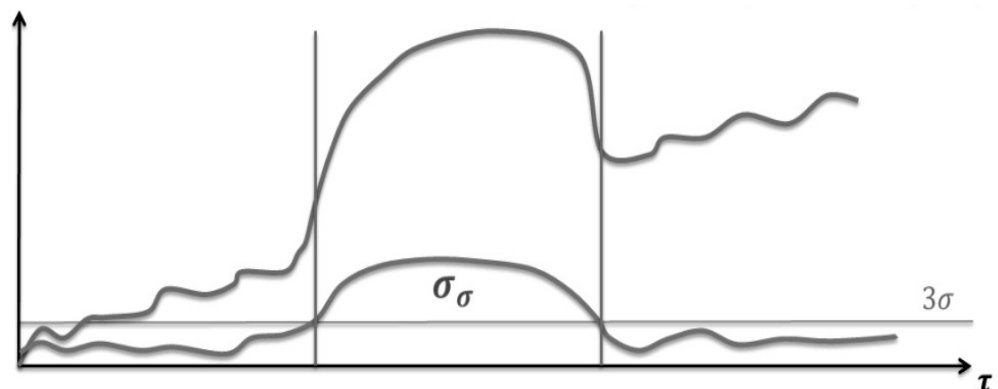


Figure 6. Crisis Model.

The Figure S1 from Supplementary File shows the graph Crisis phase identification.

During the crisis period, the intrinsic deviation of volatility exceeds the threshold value of three sigmas: $\sigma < 3\sigma_\sigma$, where σ_σ —the intrinsic standard deviation of volatility. The presence of a crisis and the direction of its development can be expressed in the interval [0;1] as Equation (5):

$$\omega_k = 0.5 + \text{sgn}(\Delta M_{T-1}) \cdot 0.25 + \text{sgn}(\Delta M_{T-1}) \cdot 0.25 \cdot \theta(3\sigma_\sigma - \sigma) \cdot \frac{3\sigma_\sigma / \sigma}{(3\sigma_\sigma / \sigma)_{max}} \tag{5}$$

where:

ω_k —weight coefficient of the crisis in the state function;

ΔM_{T-1} —change in the mathematical expectation for the period $T-1$;

σ —volatility (standard deviation) of the indicator;

σ_σ —standard deviation of volatility;
 $(3\sigma_\sigma/\sigma)_{max}$ —the highest ratio achieved in the entire historical period.

We will formulate a generalized analytical function of the current market state Equation (6):

$$W = 0.5 + \theta(|\Delta M_{T-1}| - |k_I M_{T-1}| - 2\sigma_{T-1}) \left(0.25 \cdot \theta(\sigma - 3\sigma_\sigma) \cdot \text{sgn}(\Delta M_{T-1}) \frac{|\Delta M_{T-1}|}{|\Delta M_{T-1}|_{max}} \right) + \left(\text{sgn}(\Delta M_{T-1}) \cdot 0.25 \cdot \theta(3\sigma_\sigma - \sigma) + 0.25 \cdot \theta(3\sigma_\sigma - \sigma) \cdot \text{sgn}(\Delta M_{T-1}) \frac{3\sigma_\sigma/\sigma}{(3\sigma_\sigma/\sigma)_{max}} \right), \tag{6}$$

where: W —market state function.

4.4. Asset Investment Strategy Model

We propose a portfolio model with an adaptive strategy that is optimized with regard to the market state.

Let us imagine a market strategy model as an aggregate of constituent elements Equation (7):

$$S = \sum_{i=1}^{i=m} \omega_i \tag{7}$$

where: ω_i —weight of the portfolio strategy element, m —number of strategy elements, S —aggregated estimate of portfolio strategy.

Let us define the most common elements of portfolio investment strategies:

- Profit-taking threshold;
- Aggregate portfolio risk;
- Type of portfolio diversification;
- Degree of portfolio diversification;
- Degree of portfolio dynamicity (rebalancing frequency).

Then, the portfolio strategy estimate can be represented as Equation (8):

$$S = X_{r_{fix}} + \delta_{prt} + m_{div} + k_{div} + k_{dyn} \tag{8}$$

where: $X_{r_{fix}}$ —upper profit-taking threshold; δ_{prt} —aggregate portfolio risk; m_{div} —type of diversification; k_{div} —degree of diversification; k_{dyn} —degree of portfolio dynamicity.

Development of the strategy is dependent on the market state. For the purpose of profit maximization, the strategy must be aligned with the market condition; hence, more aggressive elements of the strategy will be optimal for growing markets, while more cautious ones—for declining markets. Let us order the components of the strategy by the aggressiveness/cautiousness ratio according to the state of the market (Table 2).

Table 2. Investment portfolio strategy selection with regard to the market state.

Market State W	Profit-Taking $X_{r_{fix}}$	Portfolio Risk δ_{prt}	Type of Diversification m_{div}	Degree of Diversification k_{div}	Portfolio Dynamicity k_{dyn}
Speculative growth	Growing portfolio	Very high	Naive	Very low	Very low
Growth	Weakly bounded portfolio	High	Grouped	Low	Low
Stagnation	Bounded portfolio	Moderate	Jurisdictional	Moderate	Moderate
Decline	Highly bounded portfolio	Low	Covariant	High	High
Crisis decline	Fixed portfolio	Very low	Beta Neutral	Very high	Very high

Methods of calculating the strategy elements: let us calculate the weight coefficients of the strategy components Equation (9):

$$\forall i; \omega_i = \frac{W}{m} \quad (9)$$

where: ω —weight coefficient of the strategy component; i —the ordinal number of the strategy element; m —number of strategy elements; W —the current value of the market state.

Determination of the profit-taking threshold: to reduce the risk, we will determine the upper threshold of the portfolio profit-taking Equation (10):

$$X_{r_{fix}} = X_{p_{beg}} + W|\Delta M_{T-1}|, \quad (10)$$

where: W —the current value of the market state; $X_{r_{fix}}$ —upper profit-taking threshold of the portfolio value; $X_{p_{beg}}$ —the initial portfolio value.

Determination of the risk component of the portfolio strategy: let us define the acceptable boundaries of the risk corridor as $[\delta_{min}; \delta_{max}]$, then the recommended portfolio risk is Equation (11):

$$\delta_p = W \cdot (\delta_{max} - \delta_{min}) \quad (11)$$

where: δ_p —recommended portfolio risk; δ_{min} —risk of the most reliable asset in the portfolio; δ_{max} —risk of the least reliable asset in the portfolio.

Determination of the type of portfolio diversification: let us determine the type of diversification for the investment portfolio being formed: we will rank the types of diversification by the degree of risk increase (Table 3).

Table 3. Types of diversification by the degree of risk increase.

m_{div}	Risk	Type of Diversification
0	Minimum	Beta-neutral
1	Low	Covariant
2	Moderate	Jurisdictional
3	High	Industry-based
4,5	Maximum	Naive

$m_{div} = [0, 1, 2, 3, 4, 5]$, then the optimal type of portfolio diversification is Equation (12):

$$m_{div} = \left\lceil \frac{W}{0.2} \right\rceil \quad (12)$$

where: m_{div} —type of portfolio diversification.

Determination of the degree of portfolio diversification: let us define the boundaries of portfolio diversification as $[0; 1]$, then the recommended degree of portfolio diversification is Equation (13):

$$K_{div} = 1 - W^{\frac{1}{W}} \quad (13)$$

where: K_{div} —the degree (coefficient) of portfolio diversification.

Determination of dynamic portfolio strategy: let us express the portfolio dynamicity as the ratio of the number of rebalancing events for the period T to the maximum possible number of them, taking into account the duration of the investment period T as the maximum possible value of t Equation (14):

$$k_{dyn} = \frac{dyn}{dyn_{max}}; dyn = \frac{T}{t}; dyn_{max} = \frac{T}{\tau}; t \in [\tau; T] \quad (14)$$

Then, the recommended k_{dyn} of the portfolio is Equation (15):

$$k_{dyn} = 1 - W \tag{15}$$

The compliance of the portfolio strategy with the current market situation rules (strategy quality) can be estimated by Equation (16):

$$q_S = 1 - \left| W - \sum_{i=1}^n \omega_i \right| \tag{16}$$

where: W —the value of the current market state; ω —the value of the selected portfolio strategy element.

Table 4 shows Compliance of portfolio strategy with the current market situation.

Table 4. Compliance of portfolio strategy with the current market situation.

Market State	Weight of the Market State W	Profit-Taking ω_1	Portfolio Risk ω_2	Type of Diversification ω_3	Degree of Diversification ω_4	Portfolio Dynamicty ω_5
Speculative growth	1	0.2	0.2	0.2	0.2	0.2
Growth	0.75	0.15	0.15	0.15	0.15	0.15
Stagnation	0.5	0.1	0.1	0.1	0.1	0.1
Decline	0.25	0.05	0.05	0.05	0.05	0.05
Crisis decline	0	0	0	0	0	0

The main task for each rebalancing of the portfolio is to determine the optimal parameters of the portfolio strategy elements for the corresponding market state Equation (17):

$$\begin{cases} q_S = 1 - \left(W - \left(X_{r_{fix}} + \delta_{prt} + m_{div} + k_{div} + k_{dyn} \right) \right) \\ q_S \rightarrow \max \end{cases} \tag{17}$$

System of equations for the portfolio strategy: thus, the dependence of the optimal portfolio strategy elements on the market state can be expressed as (18):

$$\left\{ \begin{array}{l} W = 0.5 + \theta \left(|\Delta M_{T-1}| - |k_I M_{T-1}| - 2\sigma_{T-1} \right) \left(0.25 \cdot \theta (\sigma - 3\sigma_\sigma) \cdot \text{sgn}(\Delta M_{T-1}) \frac{|\Delta M_{T-1}|}{|\Delta M_T|_{max}} \right) \\ \quad + \left(\text{sgn}(\Delta M_{T-1}) \cdot 0.25 \cdot \theta (3\sigma_\sigma - \sigma) + 0.25 \cdot \theta (3\sigma_\sigma - \sigma) \cdot \text{sgn}(\Delta M_{T-1}) \frac{3\sigma_\sigma / \sigma}{(3\sigma_\sigma / \sigma)_{max}} \right) \\ X_{r_{fix}} = X_{p_{begin}} + W |\Delta M_{T-1}| \\ \delta_{prt} = W \cdot (\delta_{max} - \delta_{min}) \\ m_{div} = \left[\frac{W}{0.2} \right] \\ k_{div} = 1 - W^{\frac{1}{W}} \\ k_{dyn} = 1 - W \\ q_S = 1 - \left(W - \left(X_{r_{fix}} + \delta_{prt} + m_{div} + k_{div} + k_{dyn} \right) \right) \\ q_S \rightarrow \max \end{array} \right. , \tag{18}$$

where $X_{r_{fix}}$ and k_{dyn} are the elements of the strategy that affect the global characteristics of the portfolio—absolute risk and sensitivity, while δ_{prt} , m_{div} and k_{div} affect the asset share allocation within the portfolio.

This optimization can be performed using the stochastic gradient descent method.

4.5. The Aggregate Forecast Model for an Asset in the Market

Time series forecasting is a matter of high value and importance, so a large number of papers are devoted to this topic. Let us review the scientific papers of contemporaries.

In the paper by Wu, Y. X., Wu, Q. B., and Zhu, J. Q. (Wu et al. 2019), the authors propose a novel model based on ensemble empirical mode decomposition (EEMD) and

long short-term memory (LSTM). They obtain results that demonstrate that the hybrid approach works well even when the number of decomposition results varies.

Zhu, J., Liu, J., Wu, P., Chen, H., and Zhou, L (Zhu et al. 2019) used the ensemble empirical mode decomposition (EEMD) and optimal combined forecasting model (CFM) and obtained results that show that the proposed approach outperforms benchmark models.

Wang, M., Zhao, L., Du, R., Wang, C., Chen, L., Tian, L., and Stanley, H. E. (Wang et al. 2018) propose a novel hybrid method that uses an integrated data-fluctuation network (DFN) and several artificial intelligence (AI) algorithms, and is named the DFN-AI model. The results of their study demonstrate that the combined model performs significantly better than their corresponding single models in terms of accuracy.

Abdollahi, H. (Abdollahi 2020) proposes a novel hybrid model for crude oil price forecasting, whose focus is on improving the accuracy of prediction, taking into consideration the characteristics existing in the oil price time series. Finally, empirical results demonstrate that the proposed hybrid model outperforms other models.

The literature review has shown that the best method is the combined ensemble forecasting method. The choice of the forecasting method in the study was motivated by the fact that the ensemble forecasting method was taken as a basis.

We use an aggregate ensemble forecast to predict the value and risk of assets. We shall form such a forecast as a combination of regressions, where it should contain elements that reflect:

- Growth or decline tendencies (linear component);
- Tendencies to growth boundedness (logarithmic component);
- Seasonality and periodicity tendencies (harmonic component);
- Tendencies to the influence of prior conditions (autoregression);
- Tendencies to the influence of external factors (complex regression).

In other words, the aggregate forecast is the averaged value of the results of several regression forecasting techniques, obtained based on the confidence coefficients of the techniques in question Equation (19).

$$F(\tau) = \frac{\sum_{i=1}^{i=n} k_{d_i} f_i(\{\tau, x(\tau), \dots\})}{n}, \quad (19)$$

The confidence coefficient is the ratio of geometrically averaged correlation coefficients of a set of forecasts obtained by a certain technique to the sum of geometrically averaged correlation coefficients of all the forecasting techniques used Equation (20):

$$k_{d_i} = \frac{\left(\prod_{j=1}^{j=\lfloor \frac{2T}{T_{\text{history}}} \rfloor} \text{cor}_{\{Y_1\}\{Y_{2i}\}_j} \right)^{\frac{1}{\lfloor \frac{2T}{T_{\text{history}}} \rfloor}}}{\sum_{i=1}^{i=m} \left(\prod_{j=1}^{j=\lfloor \frac{2T}{T_{\text{history}}} \rfloor} \text{cor}_{\{Y_1\}\{Y_{2i}\}_j} \right)^{\frac{1}{\lfloor \frac{2T}{T_{\text{history}}} \rfloor}}} \quad (20)$$

Let us consider the main forecasting methods that are used to form the aggregate forecast.

1. Averaged linear forecast (Figure 7). The averaged linear forecast approaches the linear forecast asymptotically, but, unlike the linear forecast, it preserves short-term trends of changes in the function value Equation (21).

$$y_{x_{\text{fin}}+\tau} = k \cdot \left(\bar{y} + (x - \bar{x}) \cdot \frac{n \sum_{x=1}^{x=\tau} xy - \sum_{x=1}^{x=\tau} x \sum_{x=1}^{x=\tau} y}{n \sum_{x=1}^{x=\tau} x^2 - (\sum_{x=1}^{x=\tau} x)^2} \right) + (1 - k) \cdot \left(y_{x_{\text{fin}}} + \frac{\sum_{x=x_{\text{fin}}-\tau}^{x=\tau} yx}{\tau} \right) \quad (21)$$

where k is the linearity coefficient.

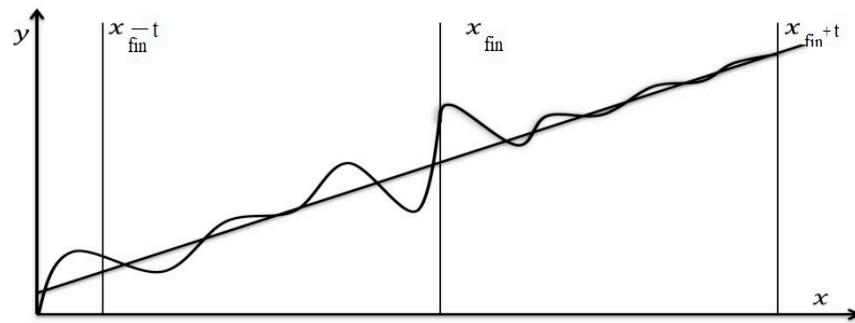


Figure 7. Averaged linear forecast.

2. Neural forecast. The neural forecast is calculated by selecting the training coefficients based on historical data of the artificial neural network using a combination of optimization (backpropagation of error) and stochastic methods.

The multilayer neural network-based forecast has the form Equation (22):

$$Y(x_\tau) = k_0 \zeta \left(\sum_{i=1}^m k_i \zeta \left(\sum_{j=i}^m k_{ij} \zeta(x_{\tau-j}) \right) \right) \tag{22}$$

$Y(x_\tau)$ —prediction function; $k_{0,1,2,i,j,ij}$ —calculated coefficients; $\zeta(x) = \frac{x}{|x|+\alpha}$ —rational sigmoid; i —the ordinal number of the neural network layer; j —the ordinal number of the neural network input; m —input width of the neural network; τ —time interval of the first predicted value Y ; x_i —main input values of the network; v_i —additional input values.

3. Combined multitrend forecast. A combined multitrend forecast represents the aggregate of linear, logarithmic, and harmonic trends, and is calculated using sequential trend decomposition.

Let us formulate the prediction function as an aggregate of logarithmic, linear, and harmonic trends Equation (23):

$$y = y_{lg} + y_{ln} + y_{gr}, \tag{23}$$

where: $y_{lg} = a_1 \cdot \ln(x) + b_1$; $y_{ln} = a_2 \cdot x + b_2$.

Thus, $y = a_1 \cdot x + b + a_2 \cdot \ln(x) + \sum a_n \cdot \sin(k_n x)$.

The aggregate forecast can be represented as Equation (24):

$$\begin{aligned} F(\tau) = & k_{d1} \cdot (k_{a1} + k_{a2} \cdot \ln(x) + k_{a3} \cdot x + \sum k_{a_n} \cdot \sin(k_{g_n} x)) \\ & + k_{d2} \cdot \left(k_{n0} \zeta \left(k_{n1} \zeta \left(\sum_{i=1}^m k_{ni} \zeta \left(\sum_{j=i}^m k_{nij} \zeta(x_{\tau-j}) \right) \right) \right) + k_{n2} \zeta \left(\sum_{i=1}^m k_{ni} \zeta \left(\sum_{j=i}^m k_{nij} \zeta(v_{\tau-j}) \right) \right) \right) \\ & + k_{d3} \cdot \left(k_l \cdot \left(\bar{y} + (x - \bar{x}) \cdot \frac{n \sum_{x=1}^{x=\tau} xy - \sum_{x=1}^{x=\tau} x \sum_{x=1}^{x=\tau} y}{n \sum_{x=1}^{x=\tau} x^2 - (\sum_{x=1}^{x=\tau} x)^2} \right) + (1 - k_l) \cdot \left(y_{x_{fin}} + \frac{\sum_{x=x_{fin}-\tau}^{x=\tau} yx}{\tau} \right) \right) \end{aligned} \tag{24}$$

4.6. The Aggregate Risk Model for an Asset in the Market

The aggregate asset risk is represented as the sum of risks Equation (25):

$$r_T = r_{b_T} + r_{n_T} + r_{o_T} \tag{25}$$

where: r_b —unconditional risk; r_n —risk of selling an asset at a loss; r_o —risk of the asset value crash.

The unconditional (stochastic) component of risk is classically described as Equation (26):

$$r_{b_T} = (D_T)^{\frac{1}{2}} \tag{26}$$

where: D_T —variance of the asset value for the period T .

The risk of selling an asset at a loss is expressed as the product of the probability of selling an asset at a price below M_i by the standard deviation of the value Equation (27).

$$r_{n_T} = P(D_T)^{\frac{1}{2}} \quad (27)$$

We estimate the probability P as the ratio of the sum of minimum time frames τ , during which the sale of an asset at a loss is possible, to total τ in T Equation (28).

$$P = \frac{\tau_{\text{loss}}}{T/\tau}, \quad P = \frac{\sum_{i=1}^{T/\tau} \theta(V_i - M_i)}{T/\tau}, \quad r_{n_T} = \frac{\tau(D_T)^{\frac{1}{2}} \sum_{i=1}^{T/\tau} \theta(V_i - M_i)}{T}, \quad (28)$$

The risk of the crash that occurs during crisis periods can be estimated based on the dynamics of the crisis, expressed as Equation (29):

$$(3\sigma_\sigma - \sigma)' \quad (29)$$

whereas in the period of crisis Equation (30):

$$\begin{cases} (3\sigma_\sigma - \sigma)' > 0; r_{o_T} = 0 \\ (3\sigma_\sigma - \sigma)' \leq 0; r_{o_T} = 1 \end{cases} \quad (30)$$

given that the maximum possible downfall will be at least 3σ , $r_{o_T} = \theta((3\sigma_\sigma - \sigma)') \cdot 3\sigma$ or $r_{o_T} = \theta((3\sigma_\sigma - \sigma)') \cdot \Delta V_{\text{max}}$, where: ΔV_{max} —the maximum drop in the asset value recorded over the historical period T ; θ —Heaviside function: $\theta(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$.

Thus, the model's aggregate risk has the form Equation (31):

$$r_T = (D_T)^{\frac{1}{2}} + \frac{\tau(D_T)^{\frac{1}{2}} \sum_{i=1}^{T/\tau} \theta(V_i - M_i)}{T} + \theta((3\sigma_\sigma - \sigma)') \cdot 3\sigma \quad (31)$$

The aggregate risk forecast r_T for an asset is calculated in the same way as the aggregate asset value forecast using a relative confidence coefficient.

Then, the problem of forming an adaptive investment strategy portfolio is formulated as follows Equation (32):

$$\begin{cases} \mathcal{E}_p \rightarrow \max \\ r_p \leq \delta_{p_s} \\ K_d \geq 1 - W^{\frac{1}{W}} \\ m_{div} = \left[\frac{W}{0,2} \right] \\ L_i \leq P_i \leq U_i \end{cases} \quad (32)$$

where: P_i —the number of assets in the portfolio; L_i, U_i —the minimum and maximum allowable number of assets; \mathcal{E}_p —portfolio efficiency; K_d —degree of diversification; W —the market state; m_{div} —type of diversification; r_p —aggregate risk of the portfolio; δ_{p_s} —maximum allowable aggregate risk.

5. Results

To compare the performance of the adaptive investment strategy with the existing classical portfolio strategies, we conducted behavioral modeling of strategic investment portfolios over a 10-year period based on real historical data for 14 years. For comparative analysis, the following types of portfolios were used:

- MPT Portfolio (by H. Markowitz) (Table 5 and Figure 8);
- PMPT Portfolio (by R. Roll) (Table 6 and Figure 9);
- Adaptive Portfolio (created by the author) (Table 7 and Figure 10).

Table 8 provides a comparison of portfolio investment performance.

These results demonstrate the dominance of the dynamic adaptive strategy not only over the static portfolio of H. Markowitz but also over the dynamic, rebalanced portfolio of R. Roll.

From the empirical data, it is evident that the dynamic adaptive strategy has a significant superiority over the strategies of H. Markowitz and R. Roll, even though it needs less rebalancing than R. Roll's strategy.

The advantage of the adaptive strategy is that it is based on predictive data as opposed to Roll's strategy, which relies on historical data only. At the same time, it also differs from H. Markowitz's strategy, where the predictive factor is solely based on the assumption that the asset price will rise in the long term.

For this reason, the main factor of the optimization and rebalancing of the value W is a leading indicator that is considerably superior to the VaR in terms of the indication and timing. In all cases, the change in the W indicator happens much earlier than the VaR extremum is reached, which allows rebalancing the portfolio proactively without waiting for the occurrence of significant losses in the portfolio value.

Table 5. Input data for H. Markowitz's portfolio.

Portfolio	Markowitz's Portfolio
Strategy	Buy and Hold
Rebalancing	By bankruptcy
Diversification limit	90.0%
Risk limit	15%
Minimum asset share	0.1%
Transaction costs	0.2%
Optimization	By rate of return
Investment term	10 years
Start of Investment	2004

Table 6. Input data for PMPT portfolio.

Portfolio	PMPT
Strategy	Periodic Rebalancing
Rebalancing	On quarterly basis
Diversification limit	90.0%
Risk limit	20%
Minimum asset share	0.1%
Transaction costs	0.2%
Optimization	By the VaR criterion
Investment term	10 years
Start of Investment	2004

Table 7. Input data for adaptive dynamic portfolio model.

Portfolio	Dynamic
Strategy	Dynamic
Rebalancing	By changes in W (market state)
Diversification limit	90.0%
Risk limit	20%
Minimum asset share	0.1%
Transaction costs	0.2%
Optimization	By W criterion (market state)
Investment term	10 years
Start of Investment	2004

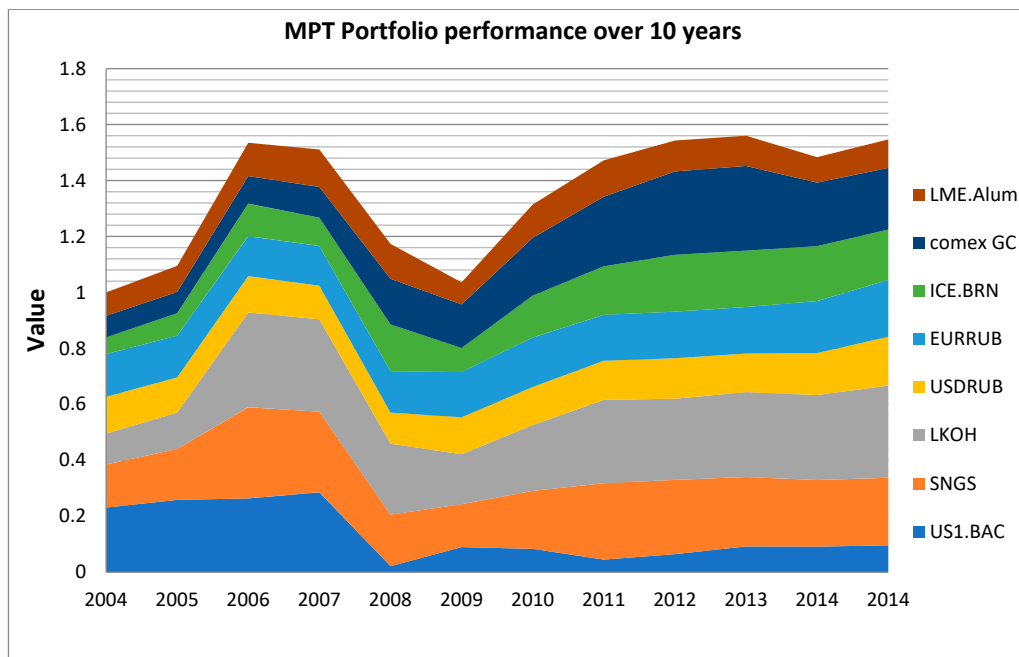


Figure 8. MPT Portfolio. Total portfolio return: 54.7%. Highest return achieved: 56%. Number of rebalancing events: 0. LME Alum—physical aluminum (contracts); Comex GC—physical gold (contracts); ICE BRN—Brent crude oil (contracts); EUR RUB—Euro (currency); USD RUB—US dollar (currency); LKOH—shares of Lukoil (shares); SNGS—shares of Surgutneftegaz (shares); US1 BAC—Bank of America shares (shares).

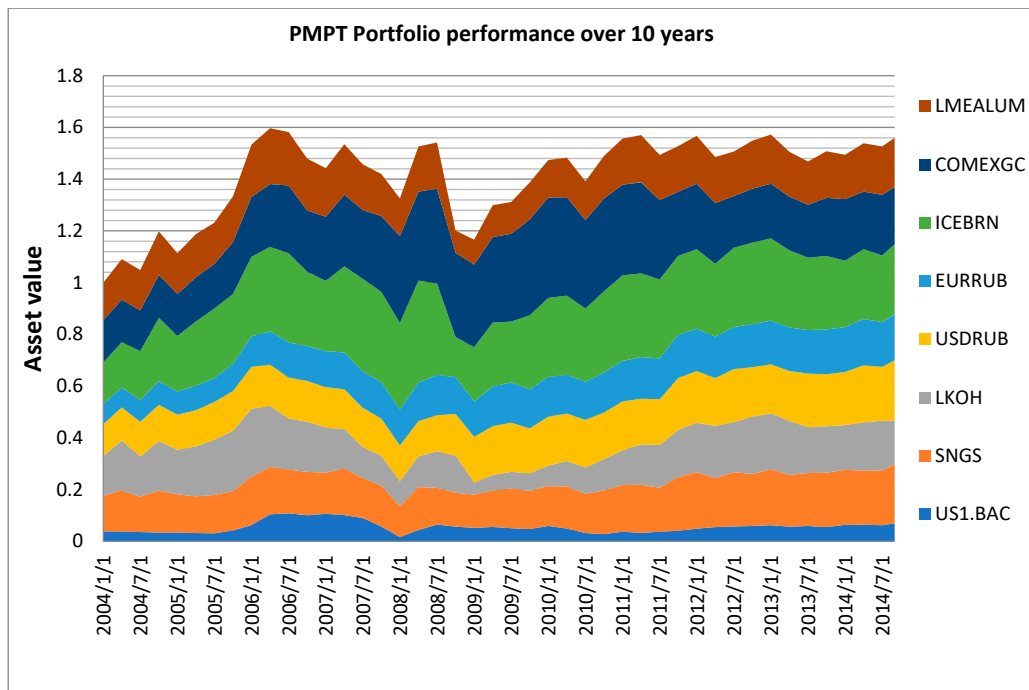


Figure 9. PMPT Portfolio. Total portfolio return: 55.9%. Highest return achieved: 59.7%. Number of rebalancing events: 40. LME Alum—physical aluminum (contracts); Comex GC—physical gold (contracts); ICE BRN—Brent crude oil (contracts); EUR RUB—Euro (currency); USD RUB—US dollar (currency); LKOH—shares of Lukoil (shares); SNGS—shares of Surgutneftegaz (shares); US1 BAC—Bank of America shares (shares).

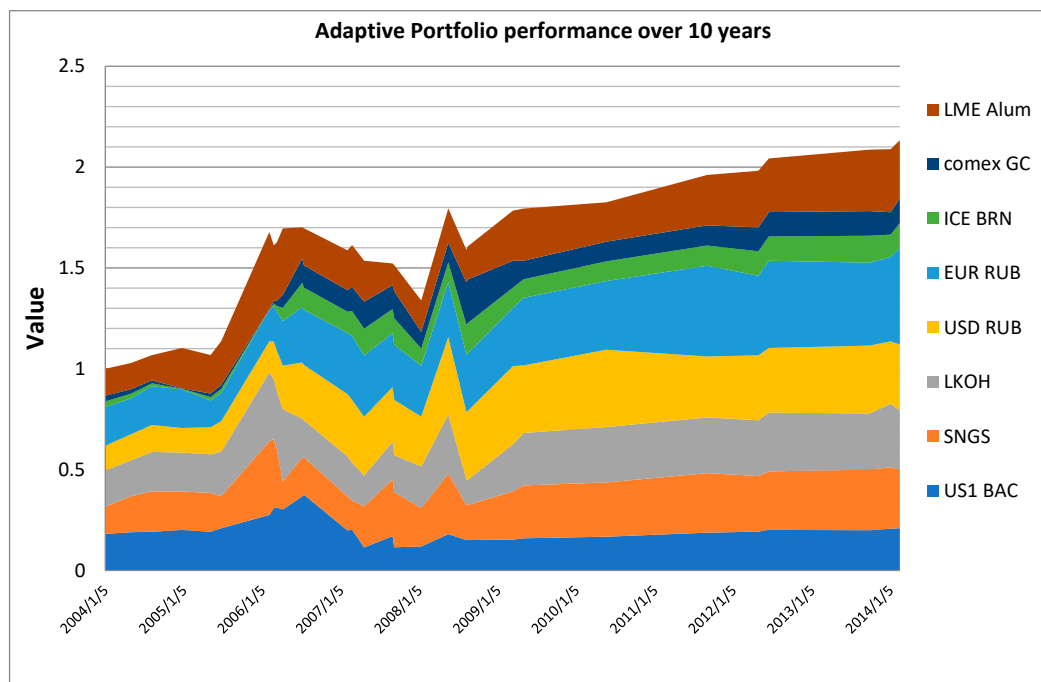


Figure 10. Adaptive Portfolio. Total portfolio return: 113.4%. Highest return achieved: 113.4%. Number of rebalancing events: 30. LME Alum—physical aluminum (contracts); Comex GC—physical gold (contracts); ICE BRN—Brent crude oil (contracts); EUR RUB—Euro (currency); USD RUB—US dollar (currency); LKOH—shares of Lukoil (shares); SNGS—shares of Surgutneftegaz (shares); US1 BAC—Bank of America shares (shares).

Table 8. Comparison of portfolio investment performance.

Portfolio Type	Average Annual Return	Number of Rebalancing Events
H. Markowitz's portfolio	5.47%	0
R. Roll's portfolio	5.59%	45
Dynamic portfolio	11.3%	30

6. Conclusions

In the time period used, the dynamic adaptive portfolio is guaranteed to have no lower return than the classic H. Markowitz's portfolio and R. Roll's portfolio. The evaluation of the movement of the asset value in the portfolio allows us to draw the following conclusions:

1. The greatest efficiency of the dynamic adaptive strategy is revealed during periods of significant market changes (crises and pre-crisis situations).

2. In tranquil market conditions, there is no need to rebalance the dynamic adaptive portfolio since the absence of significant movement in the asset value does not lead to a dramatic change in the W indicator (assessment of the state) of the asset. In this case, the dynamic portfolio will not have superiority over H. Markowitz's portfolio, but will be inferior in returns to the arbitrage portfolio due to the lack of rebalancing.

3. In times of crisis, the dynamic adaptive portfolio model demonstrates better return performance than the classic portfolio models. However, during periods of a stable market, the models of H. Markowitz and R. Roll are preferable since they do not require additional costs for data collection, processing, and forecasting, and also allow the investor to take a wait-and-see position, that is, to reduce their own time spent on portfolio management to zero.

Thus, the analysis of the results showed a comparative advantage of the dynamic adaptive strategy over models based on classical concepts.

The adaptive investment strategy allows one to respond to the changing conditions of the investment environment in time. However, it should be noted that such a strategy requires the constant attention of the investor and timely response to the signal initiation, which entails spending extra time and resources of the investor. The adaptive investment strategy allows one to reduce the risk of losing one's invested funds. This can be explained by the fact that an acceptable level of losses is set by the investors themselves, and when a given level of losses is reached, the portfolio is rebalanced, which allows the investor to save their invested capital.

Supplementary Materials: The following are available online at <https://www.mdpi.com/article/10.3390/economics9030095/s1>, Figure S1: Crisis phase identification.

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