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Application of Benford's Law to Hydrogeological Parameters: Case of the Baya Watershed (Eastern Côte d'Ivoire)

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Authors' contributions

This work was carried out in collaboration among all authors. Author MJM designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors KAK and GAD managed the analyses of the study. Author IS and JB managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

The Baya watershed is located in the eastern region of Côte d'Ivoire. It covers up a total area of 6324.041 sq. km. The exploited resources in water in that basin are surface waters and groundwaters through different drillings. The thrust of this study is to scrutinize the hydrodynamic parameters that could be influenced during drilling operation productivity in the basin through Benford statistical law. The sample is 150 drillings. This work is also the comparison of frequencies curves for parameters with regards to the empirical curve of Benford and carries out an appropriate test to see if our parameters are in line with Benford's law. The Findings obtained state that transmissivity (T) and flow (F) depend only on the natural environment. Consequently, they cannot be influenced and/or predict their values as opposed to the static level (SL), the alteration thickness (AT) and to the total depth (TD) that are not in line with that law.

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1. INTRODUCTION

Groundwaters represent 1/3, which is about 31.1% of fresh water resources of the world. It is the second reservoir of fresh waters after the glaciers and the polar cap [1]. Ground waters are contained in the aquifers and sometimes have the benefit of being protected from seasonal changes, possible and occasional accidents from pollution [2-6]. The mastery of aquifers hydrodynamic behavior containing those waters is very important for provision of drinking water for population. In this framework, various researches [7-9] have been conducted in Côte d'Ivoire about those aquifers, mainly on the Baya watershed [10] with an estimated population size of 478,327 people [11]. The work of [10] made use of various data from different hydrodynamic parameters. Those data are to be taken with care as land measurements are sometimes likely to constraints or do not seem to be done with due care and tend to be estimated, which could influence our findings. To better appreciate the drillings' productivity, the parameters used are those that will be free from any influences. Then, what would the uninfluenced parameters that will better explain hydrodynamic behavior of aquifers in the Baya watershed be? This question will bring us highlight the uninfluenced parameter using Benford's statistical law which has been used in various researches [12], namely in hydrogeology.

2. MATERIALS AND METHODS

2.1 Study Area

Baya watershed is located in the eastern region of Côte d'Ivoire between the longitudes 2°38' et 3°33' W and the latitudes 6°35' et 8°26' N. It covers a total area of about 6,324 square km (Fig. 1). The population size of the basin about 478,327 people with a rise of 2.8% of the population [11]. In terms of pedology, this basin is covered up with water table used for crops and commercial ones (coffee, cacao, cashier nut) and food crops [13]. The main geological formations can be grouped in three big lithological groups [14]. A group of tarkwaien, then volcano-sedimentary complex is mainly composed of schist, Amphibolite and of Metadolerite and an intrusive rock complex. From the hydrogeologic level, we can identify two types of aquifers within the study area. They are alteration aquifers and cracked (fissured) ones. The former is the reservoirs from physico-chemical alteration processes and from the erosion of the basement. They are composed of clayey sand and coarse-grained and they represent the first level of reservoir in the board. Those aquifers are directly operated with rainfalls. The latter are to be found in areas and/or damaged of the board. Thus, their water capacity is in relation to the density of the fracture of the board [15,16].

2.2 Material

For the purpose of this current study, we used technical data from 150 drillings (Flow (F), transmissivity (T), alteration thickness (AT), static level (SL), total depth (TD) from territorial offices for village hydraulic devices in Abengourou and Bondoukou.

2.3 Methodological Approach

Benford's law is used to highlight differences for our data about drilling in connection with seismic methods used for geological structures for sub-soil. Therefore, we will draw the curve of each parameter studied according to Benford's with reference to the ensuing equation:

$f = log(1 + \frac{1}{d})$ and then interpret them still referring to the ideal modeling of Benford (Fig. 2). That law will be applied for the transmissivity (T), Flow (F), total depth (TD), static level (SL) and to alteration thickness (AT). Data extraction was made using Excel software package. It consists in selecting the first figures, which are not null more on the left for drilling data. Thus, we were able to group the number of digits from 1 to 9 for each parameter, then calculate their frequency of occurrence and compare them to the empirical pace of the Benford curve (Fig. 2). After that, to better appreciate if the five parameters are in line with Benford's law, the statistical test Chi-square which is an appropriate test for Benford's law, was applied to these parameters.

2.4 Chi-square Test (Adequate Test for Benford's law)

Chi-square test (χ^2) is a statistical test to determine if the difference between two distributions of frequencies is due to sampling error (random) or is sufficiently important to be statistically significant. Chi-square calculated statistic (χ^2obs.) is compared with the threshold value χ^2 read on the table of χ^2 for 8 degrees of freedom in Benford's law case and for an error risk α of 10%.

• $\chi^2 obs < \chi^2$ threshold, the hypothesis H_0 (the parameter (X) follows Benford's law) cannot be rejected. The distribution of theoretical sizes and observed are not significantly different;

• $\chi^2 obs > \chi^2$ threshold, the hypothesis H_0 is rejected to the significance threshold α = 10 % and the hypothesis H_1 (the parameter (X) does not follow Benford's law) is accepted. In this study, chi-square test (χ^2) help us assert if the parameters (T), (F), (TD), (SL) and (AT)) follow Benford's law (null hypothesis Ho) or not. The chi-square test is calculated through the following formula (Eq. 1) :

$$\frac{(o_1-e_1)^2}{e_1}+\frac{(o_2-e_2)^2}{e_2}+.....+\frac{(o_p-e_p)^2}{e_p}=\sum\frac{(o_i-e_i)^2}{e_i} \quad \text{Eq. 1}$$

With

Oi = size observed
ei = theoretical size

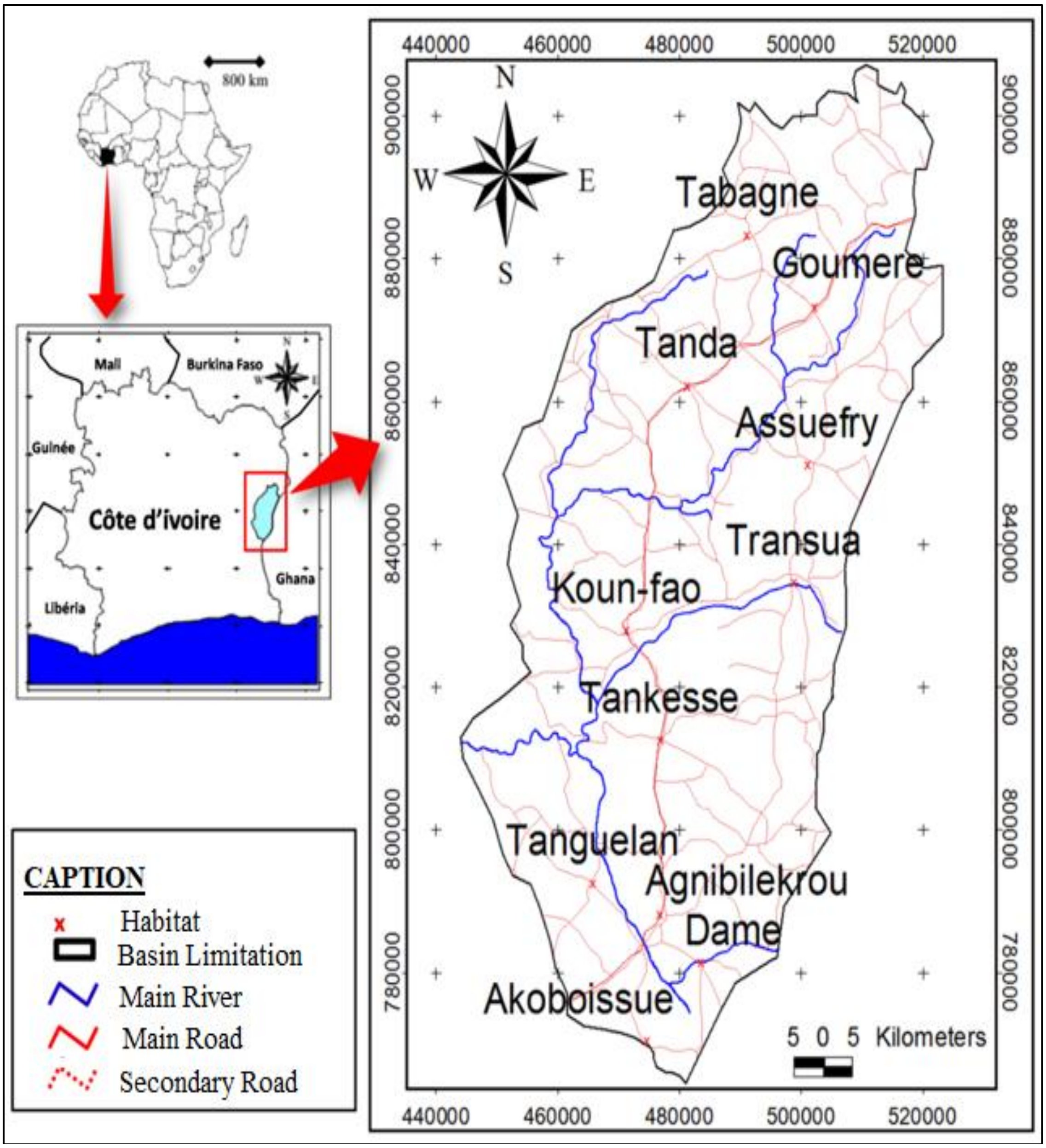

Fig. 1. Map of the study area

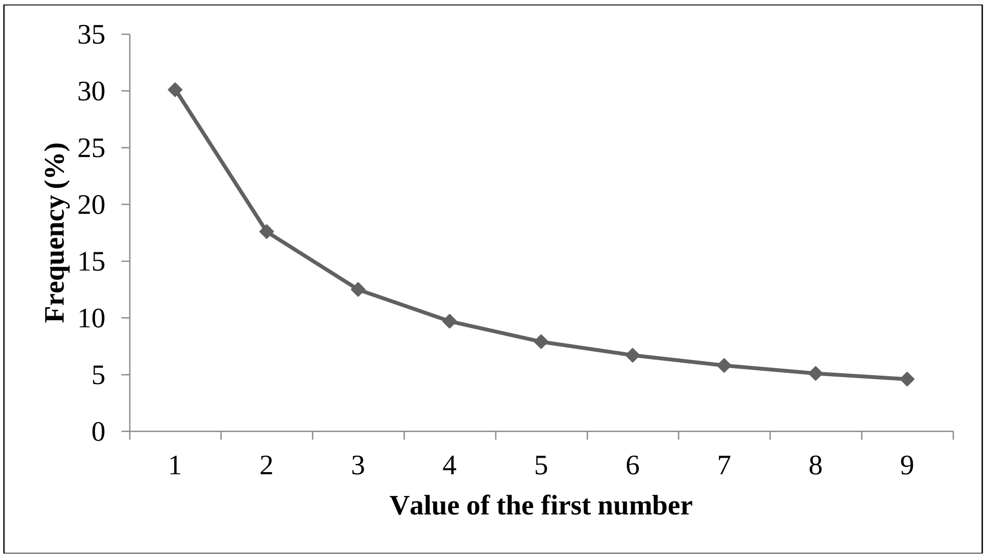

Fig. 2. Fractal parabolic distribution of Benford's law

3. FINDINGS AND DISCUSSION

3.1 Findings

The different values for the size and frequencies obtained thanks to numbering are presented in the Table1.

Fig. 3 represents frequencies curves for transmissivity (T) values, flow (F), static level (SL), total depth (TD), and alteration thickness (AT) with regards to the values of the first number from Benford's law.

The direction of the total curve depth (Fig. 3D) and the alteration thickness (Fig. 3E) are much contrasted and do not coincide with the ideal curve of Benford (curve in blue). For transmissivity (Fig. 3A), flow (Fig. 3B) and the static level (Fig. 3C), the lines represent parabolic orientations which are less in line with Benford's curve. To better appreciate if those parameters are in line with Benford's law, we made use of chi-square statistics.

3.2 Adequate Test for Benford's Law: Chi-Square

The chi-square values (χ^2) of each parameter are highlighted in Table 2. The degree of freedom is 8. The significance threshold is α = 10 %, the chi-square value threshold (χ^2threshold or $\chi^2_{0.9}$) in the table of χ^2 is 13.36 for 8 degree of freedom. The adjustment test of chi-square to Benford's law is positive for transmissivity (T) and flow (F) and negative for static level (SL) alteration thickness (AT) and total depth (TD) to the acceptance threshold level of 10 %. These results made possible the acceptance of the hypothesis Ho pour T and F and the rejection of Benford's law for the parameters AT, SL and TD (Table 2).

Table 1. Results for different parameters with regards to Benford's law

	Fb	T		F		SL		TD		AT	
D	**Freq.**	**Size**	**Freq.**	**Size**	**Freq.**	**Size**	**Freq.**	**Size**	**Freq.**	**Size**	**Freq.**
1	30.1	46	32	44	29.33	53	35.33	4	2.66	17	11.33
2	17.6	30	21	23	15.33	34	22.66	0	0	26	17.33
3	12.5	21	14	14	9.33	23	15.33	0	0	24	16
4	9.7	11	8	15	10	22	14.66	8	5.33	35	23
5	7.9	13	8	17	11	7	4.66	25	16.66	18	12
6	6.7	6	4	13	8.66	5	3.33	41	27.33	19	12.66
7	5.8	11	7	13	8.66	2	1.33	37	24.66	5	3.33
8	5.1	8	5	6	4	1	0.66	21	14	4	2.66
9	4.6	2	1	5	3.33	3	2	14	9.33	2	1.33

Fb: Benford's law; D: Figure

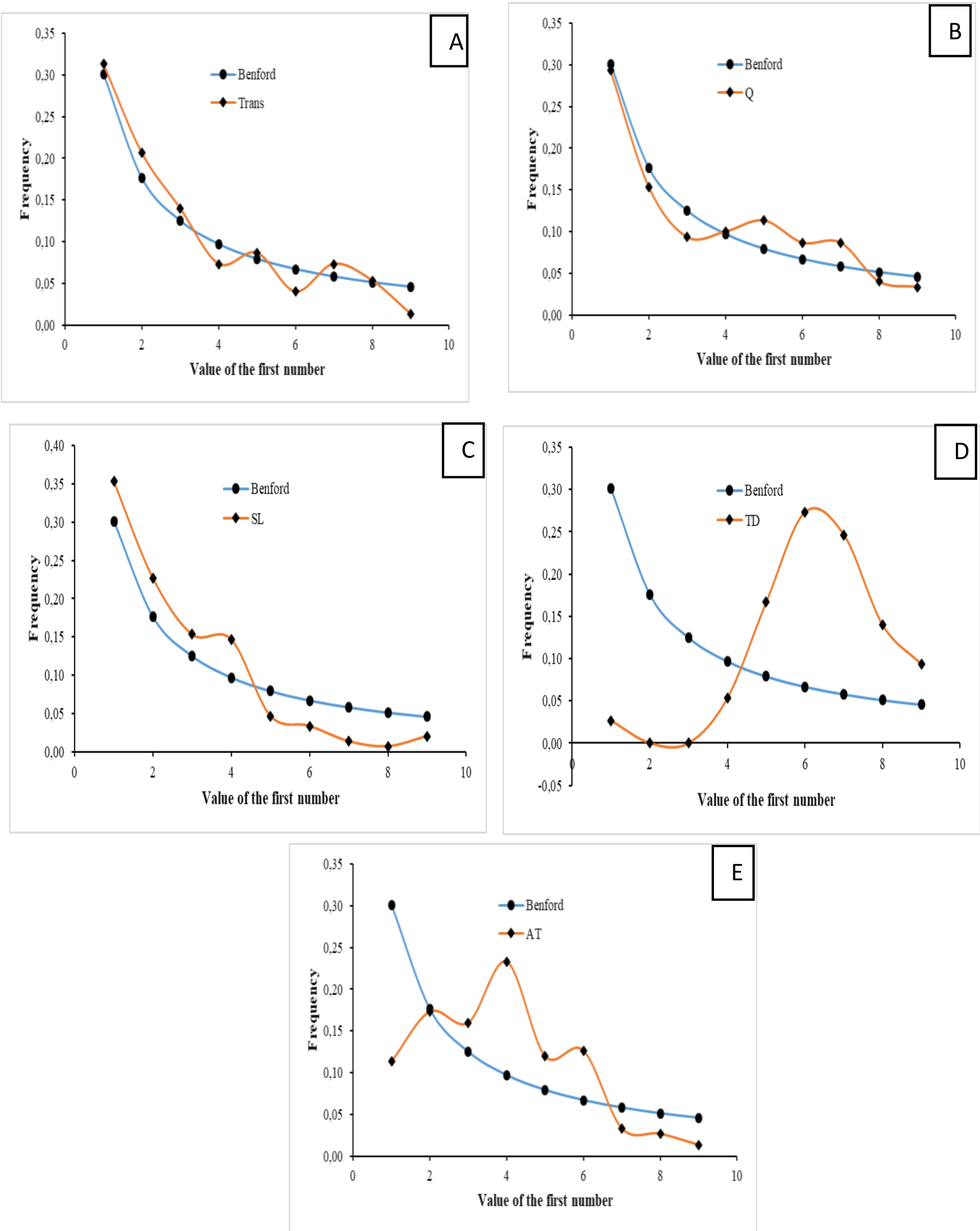

Fig. 3. Comparison of curves direction from different parameters from Benford's A)Transmissivity; B) flow; C) static level; D) depth and E) alteration thickness

Table 2. Summary of findings of chi-square for different parameters

	T	F	SL	AT	TD
δ^2_{obs}	5.41	4.91	17.34	42.90	212.04
δ^2	13.36	13.36	13.36	13.36	13.36
H_0	Accepted	Accepted	Rejected	Rejected	Rejected

3.3 Discussion

The parameters T and F are in line with Benford's law as they dependent on natural conditions of environment only [17]. As for the static level, it changes much with season, closeness to a well, a pump and even to a river as they sometimes impact water level in drilling. Measurement on ground is made when population takes water in the drilling. It could explain that inadequate situation to Benford's law. Total depth and alteration thicknesses are not in line with Benford's law in so far as they undergo financial constraints, limitations for drilling not to be crossed and/or for the appreciation of the technician during the work. On Baya watershed, TD (total depth) varies from 42.7 to 110.53 m. Therefore, it will be hard to get the first significant figures « 1 », « 2 » and « 3 ». For AT (alteration thickness), 76% of drilling have thickness sizes that vary between 25 and 70 m and 50 % that vary between 40 and 70 m. that is why, the parameters TD, SL and AT are not in line with Benford's law even if their data are spread and are regular as indicated by [12,18]. In other words, if there is no adequate situation with Benford's law, this does give any doubt about the credibility of our data. Types of hydrogeologic data in line with Benford's law must be known as posited by [19,20] for accountancy and purchasing materials. The application of Benford's law in hydrogeology helps us identify those parameters, those on which we can interact (AT, SL and TD) and the ones (F and T) on which no one can predict or foresee the value when installing a drilling.

4. CONCLUSION

The study of the hydrodynamic parameters influencing drilling productivity of the Baya watershed using Benford's law reveals that:

The parameters of transmissivity (T) and flow (F) are in line with Benford's law as they are uniquely dependent upon the natural environment. The results of ourr findings have been consistent with adequate test for chi-square which was positive;

The static level (SL) parameters, total depth (TD) and alliterate thickness (AT) do not follow Benford's law: SL is dependent upon seasonal and anthropic factors; TD and AT undergo constraints from various orders. The adequate test showed that Benford's law could not be used for those parameters, which brought us to draw early conclusion.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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