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Note on Quasi Lindley Distribution: Some Remarks and Corrections

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Short Research Article

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Abstract

Some remarks and corrections of some properties the new distribution, quasi Lindley, of which the Lindley distribution is a special case, are given concerning its parameter space. In addition, a comparison study between the new two-parameter distributions (pseudo Lindley, gamma Lindley, quasi Lindley and two-parameter Lindley) is studied.

Keywords: Lindley distribution; quasi lindley distribution; pseudo lindley distribution; goodness of fit.

1 Introduction

New distributions were proposed by Shanker and Mishra [1] and Shanker et al. [2] called the quasi Lindley distribution (QL) and the two-parameter Lindley distribution. It's are a mixture of a gamma $(2,\theta)$ and

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exponential distributions. Also, Nedjar and Zeghdoudi [3,4] called the Gamma Lindley distribution (GaL). Also, Messaadia and Zeghdoudi [5], Zeghdoudi and Nedjar [6], Beghriche and Zeghdoudi [7], Subhradev et al. [8], Zeghdoudi et al. [9] introduced another new distributions, called Zeghdoudi distribution, the pseudo-Lindley distribution which is based on mixtures of gamma $(2,\theta)$ and exponential (θ) distributions, Size Biased Gamma Lindley distribution, X gamma distribution, Lindley Pareto distribution. Shanker and Mishra [1] developed various properties of quasi Lindley distribution, such as the probability density function (pdf), survival function, cumulative distribution function (cdf) and others statistical properties. Plots of the pdf and cdf for some parameter values were also given, along with maximum likelihood estimates and moment estimates. However, there parameter space was incorrect. The density function of the random variable X was given by:

$$f_{QL}(x;\alpha,\theta)=\frac{\theta(\alpha+x\theta)}{\alpha+1}\exp(-\theta x),\alpha>-1,\theta,x>0$$

$$f_{TPL}(x;\beta,\theta)=\frac{\theta(1+\beta x)}{\beta+\theta}\exp(-\theta x),\beta>-\theta,\theta,x>0$$

Unfortunately, $f_{QL}(x;\alpha,\theta)$ and $f_{TPL}(x;\beta,\theta)$ is not a proper pdf, because each of them can be negative for some values of the parameters $\alpha>-1,\beta>-\theta$. For example, see Fig. 1. To obtain a proper pdf for $f_{QL}(x;\alpha,\theta)$, modify the parameter space to be $\theta>0$, $\alpha>0$. Now it can be shown that the proper $f_{QL}(x;\alpha,\theta)$, where $\theta>0$ and $\alpha>0$ are, in fact, general cases of a two-parameter Lindley distribution (see Shanker et al. [2]) Taking $\alpha=\frac{\theta}{\beta}$ in the pdf of the quasi Lindley distribution leads to the pdf of the two-parameter Lindley distribution, and pseudo Lindley distribution with pdf

$$f_{PsL}(x;\beta,\theta)=\frac{\theta(\beta-1+\theta x)}{\beta}\exp(-\theta x),\beta\geq 1,\theta,x>0$$

Taking $\alpha=\frac{\theta}{\beta-1}$ in the pdf of the two-parameter Lindley distribution leads to the pdf of the pseudo Lindley distribution.

Taking $\alpha=\beta-1$ in the pdf of the quasi Lindley distribution leads to the pdf of the two- parameter Lindley distribution.

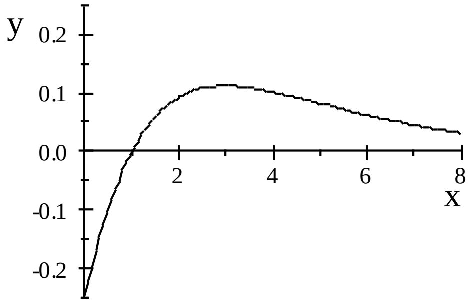

Fig. 1. Plots of the density function of QLD for α=-0.5, θ=0.5

1.1 Correctness of Some Properties of QLD

The first derivative of Equation (5) is

$$f_{QL}\left(x;\alpha,\theta\right)=\frac{-\theta^{2}}{\alpha+1}\left(\theta x+\left(\alpha-1\right)\right)\exp\left(-\theta x\right),\alpha>0,\theta,x>0$$

From this it follows that

$$\text{mode}=\begin{cases}\frac{1-\alpha}{\theta} & if \quad 0<\alpha<1\\ 0 & if \quad \alpha\geq 1\end{cases}$$

The cumulative distribution function of the quasi Lindley distribution is obtained as

$$F\left(x;\alpha,\theta\right)=\frac{1+\alpha+\theta x}{1+\alpha}\exp\left(-\theta x\right),\alpha>0,\theta,x>0$$

1.1.1 Estimates from moments of QLD

$$\mu_{r}=\frac{r!}{\theta^{r}\left(\alpha+1\right)}\left(\alpha+r+1\right)$$

$$m_{1}=\frac{\alpha+2}{\theta\left(\alpha+1\right)}$$

$$m_{2}=\frac{2\left(\alpha+3\right)}{\theta^{2}\left(\alpha+1\right)}$$

$$k\left(\alpha\right)=\frac{m_{2}}{m_{1}^{2}}=\frac{2\left(\alpha+1\right)\left(\alpha+3\right)}{\left(\alpha+2\right)^{2}}$$

$$\left(k-2\right)\alpha^{2}+\left(4k-8\right)\alpha+\left(4k-6\right)=0$$

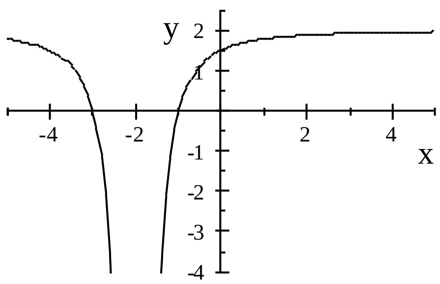

Fig. 2. plot of k(α) function

If $\frac{3}{2}\leq k\leq 2$

Then

$$\alpha = -\frac{1}{k-2}\left(2k+\sqrt{2}\sqrt{-k+2}-4\right)$$
$$\widehat{\theta} = \frac{\alpha+2}{m_1\left(\alpha+1\right)}$$

1.1.2 Goodness of fit

Table 1. Data of survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal (1960))(see Shanker and Mishra [1])

Survival time (in days)	**0-80**	**80-160**	**160-240**	**240-320**	**320-400**	**400-480**	**480-560**	**Total**
Observed frequency	8	30	18	8	4	3	1	72

According Table 1, the parameters QLD estimation α *and* θ are -0.2609, 0.013 respectively. Moreover, the first and second moments are

$$m_1 = 180.33$$
$$m_2 = 43627.78$$
$$\frac{m_2}{m_1^2} = 1.34 \notin \left[\frac{3}{2}, 2\right]$$

Consequently, it is impossible to approximate these data using the QLD.

Table 2. Comparison between distributions

Data	Distribution	β	θ	γ	$-LL$	*KS*	*AIC*	*BIC*
*Data*1								
	PsL	1.063	0.0684		**62.075**	**0.082**	**128.15**	**129.57**
$n=15$	*GaLD*	1.129	0.0684		64.015	0.094	132.03	133.45
$m=27.546$	*QLD*	4.016	*not applicable*		1504	0.93	3012	3013.4
$s=20.06$	*TwoPLD*	0.070	1.110		---	0.196	---	---
Data 2								
	PsL	1.086	0.010		**150.232**	**0.128**	**304.464**	**306.9**
$n=25$	*GaL*	0.05	0.010		152.132	0.129	308.26	310.7
$m=178.32$	*QLD*	0.010	8.514		1045.9	0.94	2131.8	2156.2
$s = 131.1$	*TwoPLD*	0.010	0.125		---	0.232	---	---

1.2 Data applications

In this section, we give a comparison study between the new two parameters distributions *(pseudo Lindley, gamma Lindley, quasi Lindley and two -parameter Lindley).* For this, we consider two data sets.

According to Table 2, we can observe that pseudo Lindley distribution provide smallest -LL, AIC, and BIC values as compared to gamma Lindley, quasi Lindley, two parameters Lindley distributions, and therefore best matches the data among all models considered.

2 Conclusion

This is not a criticism of Shanker and Mishra's novel contribution. Some remarks and correction on quasi Lindley distribution is given pertaining to its parameter space. The quasi Lindley distribution might attract wider sets of applications in actuarial science, finance, medicine, and engineering. The reliability behavior of the quasi Lindley distribution allows an improved performance for lifetime data modeling, and the hazard rate function can have various shapes, so this approach is more realistic and provides a greater degree of flexibility. Also, a new version of a compound Poisson distribution named the Poisson quasi Lindley (PQL) distribution (see [10]) was obtained by compounding the Poisson and quasi Lindley distributions, which is applicable to the collective risk model in no life insurance.

Competing Interests

Authors have declared that no competing interests exist.

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