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Analysis of Transmission Dynamics of Anthrax in Animals: A Modeling Approach

George Thiong'o Githire[1*], George Kimathi[1] and Mary Wainaina[1]

[1]Department of Mathematics and Actuarial Sciences, Catholic University of Eastern Africa, Kenya.

Authors' contributions

This work was carried out in collaboration among all authors. Author GTG designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors GK and MW managed the analyses of the study. Author GTG managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

This paper seeks to develop a SIR model with vaccination compartment in the study of anthrax transmission dynamics in animal population. The model employ ordinary differential equations in the formulation of its equation. The model's steady states solutions are investigated. The disease free equilibrium and endemic equilibrium of the model are analyzed qualitatively. Vaccination rate below a certain critical value causes the anthrax disease to persist. Recruitment and contact rates are the most sensitive parameters that significantly contribute to the basic reproductive ratio.

Keywords: Anthrax; disease; vaccination; SIR model; endemic equilibrium.

1. INTRODUCTION

Anthrax is an infectious disease categorized under zoonotic diseases caused by a bacterium called *Bacillus anthracis* [1]. It is a disease that affects both animal and human population. This disease is found naturally in soil [2] and mostly affects herbivores as compared to carnivores. Anthrax is one of the major diseases that cause uncontrolled mortality in cattle, pigs, sheep,

**Corresponding author: E-mail: githiregeorge@gmail.com;*

goats and horses worldwide [3,4,5,6]. Animals get infected with anthrax through contact with infected animals, consumption of infected grass or water and by inhalation of its spores [7]. The environment is usually infected by carcasses of infected animals. Grass and soil become the reservoirs of anthrax spores which can persist in the soil or grass for an extended period of time even under very extreme weather and environmental conditions. According to authors [8,9,10], transmission of anthrax in animals can be done through vaccination. This can be achieved easily by treating infected animals or vaccinating those joining the herd or recovered ones. Treated animals do acquire temporal immunity. Therefore, there is need to vaccinate all live animals against anthrax disease to prevent transmission.

The infective animals will show clinical symptoms which take time to manifest because the incubation period of anthrax is about 3-8 days before they succumb to death.

Authors in [11] model considers four compartments: Susceptible, Contamination, Infective and Pathogens. This model regards infective compartment level key to the transmission of anthrax in animals. In this model, the infected animals do show clinical symptoms of the disease that can be transmitted to susceptible animals. This model is an extension of work done by author [12]. According to author, the model considers three compartments namely: Susceptible, Contamination and Pathogens. The model does not consider infective compartment to be key in the transmission of anthrax disease. According to the model, the infective compartment is considered to have very low reproductive ratio [1,13] and does not cause any infections in animals.

According to research done by authors in [14], the model considers four compartments: Transmission, Carcass ingestion, Environment and Migration as possible means through which anthrax is transmitted in animals. In this model, carcass ingestion and removal of carcasses from the environment does not result to decline of anthrax transmission in animals. This model is an extension of work done previously by authors [15] whose model only considers two compartments: Environment and Contamination. In the study done by authors [4], their model considers seven compartments. This model studies the transmission dynamics of anthrax disease between animal and human population. Their model considers sensitivity analysis and how each of the parameters contributes on the model.

2. THE MODEL

In our study model, we have employed part of the model constructed by authors [4] to investigate the effectiveness of constant vaccination policy on SIR model. Constant vaccination policy is an effective disease control mechanism that ensures that all animals joining the herd are vaccinated to prevent the transmission of the disease. This model also seeks to address the impact of recruitment and contact rates in the transmission dynamics of anthrax in animals. This was found not to have been captured by authors [4]. This model also investigates the impact of disease-induced death rate in the animal population. However, this model cannot fully guarantee that all intruding and susceptible animals are vaccinated because of their large numbers and that the animal population interaction is not in a closed system.

This model was derived from the fact that in most of our game reserves and national parks, human involvement in the transmission of anthrax is minimal though there are cases of intrusion by poachers killing animals for hide, skin and tusks. Assuming human contribution to be negligible, it becomes therefore significantly important to investigate the transmission of anthrax disease in animal population only.

Our framework considers four compartments: Susceptible, Infective, Recovered and Vaccinated. The total animal population is divided at any time (t) into the four compartments with respect to their disease status in the system. The total animal population is given by N (t) =S(t)+I(t)+R(t)+V(t) where S (t) represents animals at risk of developing anthrax infection, I (t) represents animals showing anthrax symptoms, R (t) represents animals recovered from anthrax infection and acquired temporal immunity and V (t) represents animals vaccinated against anthrax attack.

The parameters used in this model are: λ denotes recruitment rate; β denotes contact rate; μ denotes natural death rate; γ denotes vaccination rate; τ denotes waning immunity of vaccinated animals; σ denotes waning recovery rate; θ denotes disease induced death rate and

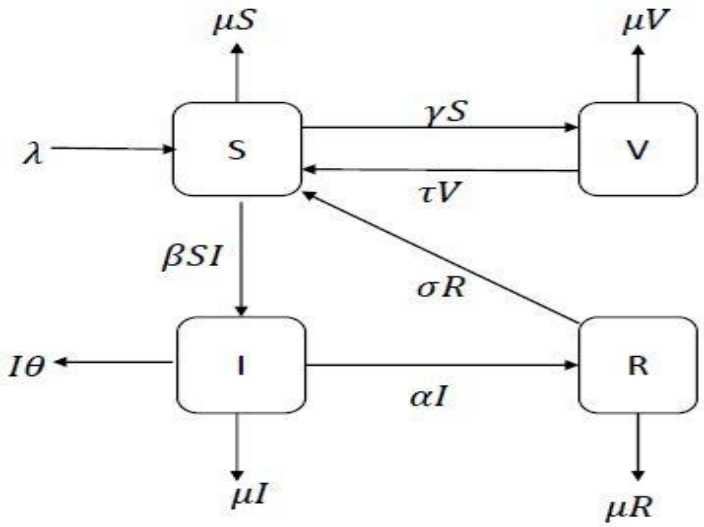

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Fig. 1. SIR Flow chart with vaccination compartment

α animal recovery rate. These variables and parameters used are all non-negative part.

Fig. 1 shows SIR model flow chart with vaccination compartment for anthrax transmission in animal population.

The model equations are non-linear ordinary differential equations given by:

$$\frac{dS}{dt} = \lambda - \beta SI - (\mu + \gamma)S + \sigma R + \tau V \quad (1)$$

$$\frac{dI}{dt} = \beta SI - (\mu + \theta + \alpha)I \quad (2)$$

$$\frac{dR}{dt} = \alpha I - (\mu + \sigma)R \quad (3)$$

$$\frac{dV}{dt} = \gamma S - (\mu + \tau)V \quad (4)$$

Equation (1) describes the dynamics of the susceptible animals. Equation (2) describes the nature of infected animals from anthrax disease. Equation (3) describes the dynamics of recovered animals after undergoing treatment and equation (4) describes the dynamics of vaccinated animals with anthrax vaccine.

3. DISEASE FREE EQUILIBRIUM

Disease Free Equilibrium is given by $\varepsilon^0 = (S^0, I^0, R^0, V^0)$. At Disease Free Equilibrium there exists no infection, no recovery. According to authors in [16,17], the dynamical systems of ordinary differential equations (1)-(4) are equated to zero to determine the equilibrium point.

$$\lambda - \beta SI - (\mu + \gamma)S + \sigma R + \tau V = 0 \quad (5)$$

$$\beta SI - (\mu + \theta + \alpha)I = 0 \quad (6)$$

$$\alpha I - (\mu + \sigma)R = 0 \quad (7)$$

$$\gamma S - (\mu + \tau)V = 0 \quad (8)$$

From (7), $I = R = 0$ at Disease Free Equilibrium.

From (6), S≠0

From (8), $\gamma S - (\mu + \tau)V = 0$ it implies that $V \neq 0$

Therefore, $V(t) = \frac{\gamma S(t)}{\mu + \tau}$

From (5), $\lambda - (\mu + \gamma)S + \tau(\frac{\gamma S}{\mu + \tau}) = 0$

Therefore $S^0 = \dfrac{\lambda(\mu+\tau)}{\mu^2+\mu\gamma+\mu\tau}$

$$S^0 = \frac{\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)} \qquad V^0 = \frac{\lambda\gamma}{\mu(\mu+\gamma+\tau)}$$

The Disease Free Equilibrium point becomes $\varepsilon^0 = \left(\dfrac{\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)}, 0, 0, \dfrac{\lambda\gamma}{\mu(\mu+\gamma+\tau)}\right)$

4. BASIC REPRODUCTIVE RATIO R_0

According to authors in [1,13], the basic reproductive ratio (R_0) can be found using Jacobian matrix J of ordinary differential equations (1)-(4) differentiated partially to yield:

$$\text{J (SIRV)} = \begin{bmatrix} -(\mu+\gamma) & -\beta\left[\dfrac{\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)}\right] & \sigma & \tau \\ 0 & \beta\left[\dfrac{\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)}\right]-(\mu+\theta+\alpha) & 0 & 0 \\ 0 & \alpha & -(\mu+\sigma) & 0 \\ 0 & 0 & 0 & -(\mu+\tau) \end{bmatrix} \tag{9}$$

From the Jacobian matrix, one of the eigenvalues is $-(\mu+\gamma)$ .The rest of the eigenvalues can be found using matrix A given by:

$$\text{A} = \begin{bmatrix} \beta\left[\dfrac{\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)}\right]-(\mu+\theta+\alpha) & 0 & 0 \\ \alpha & -(\mu+\sigma) & 0 \\ 0 & 0 & -(\mu+\tau) \end{bmatrix} \tag{10}$$

The determinant of A $= \left[\dfrac{\beta\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)}-(\mu+\theta+\alpha)\right](\mu+\sigma)(\mu+\tau)$

This determinant can be represented as

$$\left[\frac{\beta\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)(\mu+\theta+\alpha)}-1\right](\mu+\sigma)(\mu+\tau) \tag{11}$$

From (11), the basic reproductive ratio

$$R_0 = \frac{\beta\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)(\mu+\theta+\alpha)} \tag{12}$$

Theorem 1:

Disease free equilibrium point is locally asymptotically stable if R_0 < 1 and is unstable if R_0 > 1.

Proof:

Disease free equilibrium point given as $\varepsilon^0\left(\frac{\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)},0,0,\frac{\lambda\gamma}{\mu(\mu+\gamma+\tau)}\right)$ and basic reproductive ratio given as $R_0=\frac{\beta\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)}-(\mu+\theta+\alpha)$

At disease free equilibrium

$$\frac{\beta\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)}-(\mu+\theta+\alpha) < 0 \quad (13)$$

Equation (13) can be expressed as

$$\frac{\beta\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)(\mu+\theta+\alpha)}-1 < 0. \quad (14)$$

Using (12) in (14) yields R_0-1 < 0 implying R_0 < 1.

Given that R_0 < 1, disease free equilibrium point exists and locally asymptotically stable.

Lemma 1:

If R_0 >0, then it follows that $\frac{\beta\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)}-1$ > 0.

Therefore, R_0 > 1 which implies that the disease free equilibrium is locally asymptotically unstable.

5. ENDEMIC EQUILIBRIUM

The Endemic Equilibrium state is where the anthrax disease persists and cannot be eradicated from animal population. The susceptible, infected, recovered and vaccinated compartments must not be zero at this equilibrium point. According to authors in [18], endemic equilibrium of dynamical systems (1)-(4) is given by $\varepsilon^*=(S^*,I^*,R^*,V^*)$ where S^* >0, I^* >0, R^* >0 and V^* >0.

From (7), the rest point becomes:

$$S^*=\frac{\mu+\theta+\alpha}{\beta}, \qquad I^*=\frac{(\mu+\gamma)(\mu+\tau)(\mu+\theta+\alpha)-\gamma\tau(\mu+\theta+\alpha)-\beta\lambda(\mu+\sigma)}{\beta(\mu+\tau)\left[\sigma\tau-(\mu+\sigma)(\mu+\theta+\alpha)\right]}(\mu+\sigma) \quad (15)$$

$$R^*=\tau\frac{(\mu+\gamma)(\mu+\tau)(\mu+\theta+\alpha)-\gamma\tau(\mu+\theta+\alpha)-\beta\lambda(\mu+\tau)}{\beta(\mu+\tau)\left[\sigma\tau-(\mu+\sigma)(\mu+\theta+\alpha)\right]}, \qquad V^*=\gamma\frac{(\mu+\theta+\alpha)}{\beta(\mu+\tau)}$$

Theorem 2:

If $\gamma < (\mu+\tau)\left(\frac{\beta\lambda}{\mu(\mu+\theta+\alpha)}-1\right)$, the vaccination threshold less than a certain critical value, the endemic equilibrium point becomes unstable.

Proof:

From (9) the Jacobian matrix J (SIRV) =

$$\begin{bmatrix} -(\mu+\gamma) & -\beta\left[\frac{\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)}\right] & \sigma & \tau \\ 0 & \beta\left[\frac{\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)}\right]-(\mu+\theta+\alpha) & 0 & 0 \\ 0 & \alpha & -(\mu+\sigma) & 0 \\ 0 & 0 & 0 & -(\mu+\tau) \end{bmatrix}$$

If the determinant is greater than zero,

Then

$$\frac{\beta\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)} - (\mu+\theta+\alpha) > 0 \quad (16)$$

Re-arranging (16) yields

$$\frac{\beta\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)(\mu+\theta+\alpha)} - 1 > 0 \quad (17)$$

From (12), $R_0 = \frac{\beta\lambda(\mu+\tau)}{\mu(\mu+\gamma+\tau)(\mu+\theta+\alpha)}$

Therefore,

$$R_0 > 1 \quad (18)$$

The endemic equilibrium will only occur if R_0>1. This means that the disease become unstable and the rest point is lost. The vaccinated animal lose their immunity to become susceptible.

Lemma 2:

If $\gamma > (\mu+\tau)(\frac{\beta\lambda}{\mu(\mu+\theta+\alpha)} - 1)$, the endemic equilibrium point becomes stable. Anthrax disease die in animal population.

6. SENSITIVITY ANALYSIS AND BASELINE VALUES

Sensitivity Analysis is used in determining how the parameters contribute to the basic reproductive ratio R_0 in the model. The table below shows sensitivity index and baseline values of each parameter and how it contributes to the model. Sensitivity analysis is given by the relation:

$S_A^{R_0} = \frac{\partial R_0}{\partial A} \times \frac{A}{R_0}$ .Where A is any parameter used on the model.

7. RESULTS

In order to study this model, we take the initial conditions for endemic equilibrium $\varepsilon^*($ $S^* = 2000, I^* = 100, R^* = 300, V^* = 500)$ over a period of 10 years. Parameters baseline values used are from published literature on anthrax disease as indicated above. These baseline values from other published literature have been employed into this model for they have been tested in their models and given results. Where there is no data for the parameters an estimation data is given after running some tests. A Matlab software (odesolve) has been used to run the tests and graphs are obtained as depicted in Figs. 2a-d.

Fig. 2a shows that the susceptible animals are highly infected by anthrax and the transmission of the disease remains high. Significant number of animals die due to anthrax outbreak because the vaccination rate is very low. This graph show evidence of endemic equilibrium state. The basic reproductive ratio R_0 >1.

In Fig. 2b, the vaccination rate is increased from 0.10 to 0.40 .Due to this increase, the number of susceptible animals decrease significantly. The basic reproductive ratio R_0> 1 evidence of endemic equilibrium.

In Fig. 2c, the vaccination rate is increased from 0.40 from Fig. 2b to 0.60.The number of susceptible animals decrease further as vaccination rate in increased. The infective animals remains very low. The basic reproductive ratio R_0 < 1.This is evidence of disease free equilibrium.

Fig. 2d shows further decrease in the number of susceptible animals due to increased rate of vaccination. The number of infective animals remains relatively very low. This graph show evidence where anthrax in animals is eradicated.

Table 1. Sensitivity index and baseline values of each parameter

Parameter	Contribution	Baseline values	References
λ	Positive	200	[19]
β	Positive	0.0001	[15,11]
μ	Positive	0.001	Estimate
γ	Negative	0.10	[15]
σ	Nil	0.02	Estimate
τ	Positive	0.003	[4]
θ	Negative	0.15	[11]
α	Negative	0.01	[15]

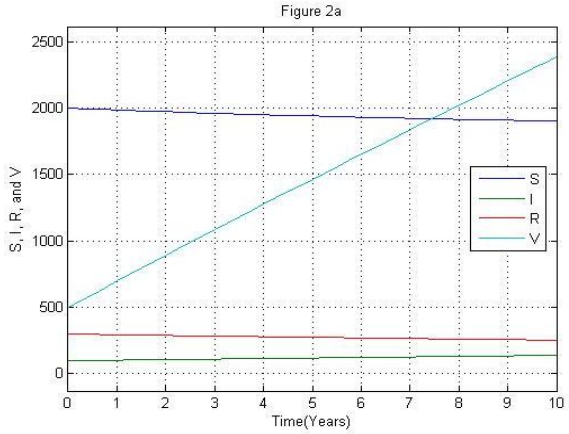

Fig. 2a. Graphical illustration for parameter values $\lambda = 200, \beta = 0.0001, \mu = 0.001$ $\gamma = 0.1, \sigma = 0.02, \tau = 0.003, \theta = 0.15, \alpha = 0.01$ $R_0 =$ **4.7780**

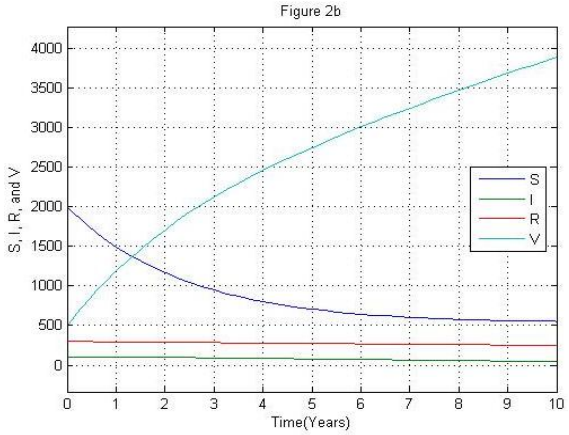

Fig. 2b. Graphical illustration for parameter values λ=200, β=0.0001, μ=0.001 γ=0.40, σ=0.02, τ=0.003, θ=0.15, α=0.01. $R_0 =$ **1.230**

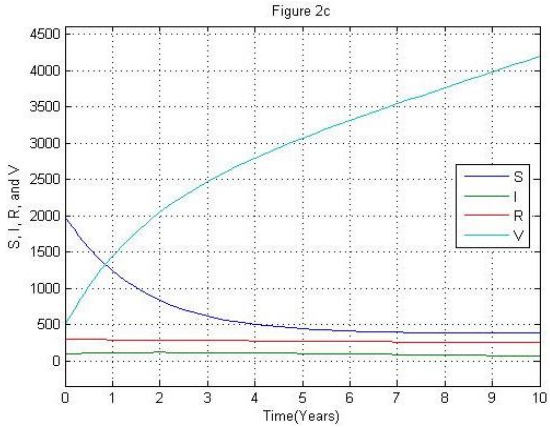

Fig. 2c. Graphical illustration for parameter values λ=200, β=0.0001, μ=0.001, γ=0.60 $\sigma = 0.02, \tau = 0.003, \theta = 0.15, \alpha = 0.01$ $R_0 =$ **0.822**

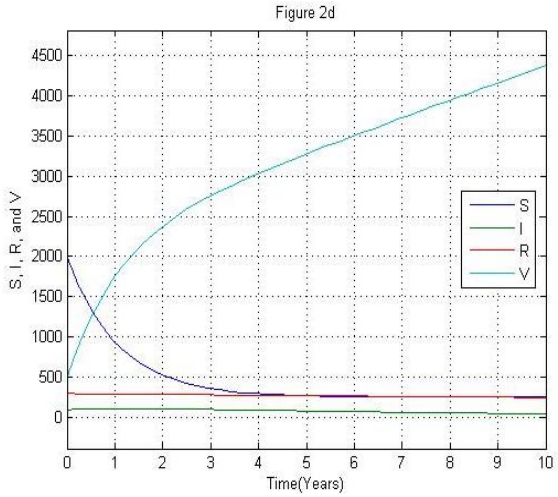

Fig. 2d. Graphical illustration for parameter values λ=200, β=0.0001, μ=0.001, γ=0.90 $\sigma = 0.02, \tau = 0.003, \theta = 0.15, \alpha = 0.01$ R_0 **=0.550**

8. DISCUSSION

In this study, we modeled vaccination compartment in the transmission dynamics of anthrax in animal population. The outcome of stability analysis of the endemic equilibrium state shows that it is possible to effectively control anthrax transmission in animal population by vaccinating the animals. Our model shows that by decreasing vaccination rate would cause an increase in basic reproductive ratio. This therefore imply that more animals will die due to widespread of anthrax infection.However,by increasing vaccination rate, the basic reproductive ratio decrease indicating no animals die from anthrax infection.

From assumptions made in this model, the immunity of the animals is considered temporal. Therefore with time this immunity wanes and the animals become susceptible to anthrax disease. Increasing vaccination rate cause a decline in the susceptible animals to contract the disease. Therefore, transmission of anthrax amongst the animals become significantly low. The effect of infection rate can be seen on the susceptible and infected population. The susceptible and infected population increases if the infection rate increases and vice versa. But if vaccination is introduced in the susceptible population, the infected population suddenly decrease to very low value. Therefore, transmission of anthrax in animals become low and the disease can be eradicated.

9. CONCLUSION AND RECOMMENDATIONS

The outcome of the model shows that vaccination is a good control strategy against anthrax outbreak in animal population. However, one vaccination may not completely guarantee protection of the animals against anthrax as it is possible that vaccinated animals may lose immunity and contract anthrax disease again. Therefore, there is need to keep vaccinating animals periodically against anthrax to keep anthrax transmission as low as possible.

This model is recommended to the Ministry of Agriculture, Livestock and Fisheries, government agencies and other policy makers in the management of animal diseases such as anthrax.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Castillo-Chavez C, et al. Computation of R_O and its role in global stability. In: Castillo-Chavez C et al (Eds) Mathematical approaches for emerging and re-emerging Infectious diseases: An introduction. IMA. 2002;125:229-250.

2. Wendy CT. Soil ingestion, nutrition and seasonality of anthrax in herbivores of Etosha National Park, Ecosphere. 2013; 4(1):1-19.
3. Anderson RM, May RM. Population biology of infectious diseases 1. Nature. 1979;280: 361-367.
4. Osman S, Oluwole DM, Theuri DM. Mathematical model of transmission dynamics of anthrax in human and animal population. 2018;8(6):47-67.
5. World Health Organization Report. Anthrax in humans and animals, 4th edition, World Health Organization. 2008;10.
6. World Health Organization Report. Anthrax in Humans and animals. Geneva: International Office of Epizootics, World Health Organization; 2016.
7. Bradford Gutting, et al. Mathematically modeling inhalational anthrax. Microbe. 2008;3(2):78-85.
8. Mushayabasa S. Impact of vaccination and culling on controlling foot and mouth disease: A mathematical modeling approach. World J Vaccines. 2011;1:156-161.
9. Sudipa C, Misra OP, Dhar J. Stability Analysis of SIR model with vaccination. 2014;17-23.
10. Yusuf TT, Benyah F. Optimal control of vaccination and treatment for an SIR epidemiological model. World Journal of modeling and Simulation. 2012;8(3):194-204.
11. Zerihun MS, Narasimha MS. Modeling and simulation study of anthrax attack on environment. Journal of multidisciplinary Engineering and Technology (JMEST). 2016;3:4.
12. Mushayabasa S. Global stability of an anthrax model with Environmental decontamination and time delay. Discrete Dynamics in Nature and Society; 2015. Article ID 573146.
13. Pauline van den D, Watmough J. Reproduction Numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Journal Mathematical Biosciences. 2002;180(1-2): 29-48.
14. Friedman A, Yakubu AA. Anthrax epizootic and migration: Persistence or extinction. Mathematics Bioscience. 2013;241:137-144.
15. Hahn BD, Furniss PR. A deterministic model of anthrax epizootic threshold results. Ecological Modeling. 1979;20(2-3): 233-241.
16. Castillo-Chavez C, et al. Dynamical models of Tuberculosis and their applications. Math Biosciences and Engineering. 2004; 1(2):361-404.
17. LaSalle JS. The stability of dynamical systems. CBMS-NSF regional conference series in applied mathematics. SIAM, Philadelphia. 1976;25.
18. Kalu A, Ugwa, Agwu IA, Akuagwu AN. Mathematical Analysis of the Endemic Equilibrium of the transmission Dynamics of Tuberculosis. 2013;2(12):263-269.
19. Mushayabasa S, Marijani T, Masocha M. Dynamical Analysis and control strategies in modeling anthrax. Computational and Applied Mathematics. 2015;1-16.

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